

TABLE OF CONTENTS

	Page
CHAPTER 7. BASIC PRINCIPLES	7-1
7.1 OPEN CHANNEL AND ITS PROPERTIES	7-1
7.1.1 Types of Open Channel.....	7-1
7.1.2 Types of Flow	7-5
7.2 DIMENSIONALITY.....	7-7
7.3 CONSERVATION OF MASS.....	7-8
7.4 CONSERVATION OF ENERGY.....	7-9
7.4.1 Hydrostatic Pressure Distribution.....	7-10
7.4.2 Specific Energy.....	7-12
7.4.3 Critical Flow and the Froude Number.....	7-14
7.4.4 Subcritical and Supercritical Flows	7-16
7.4.5 Applications of the Energy Principle.....	7-16
7.5 CONSERVATION OF MOMENTUM	7-17
7.5.1 The Momentum Function	7-19
7.5.2 The M-y Relationship.....	7-19
7.6 FLOW RESISTANCE	7-21
7.6.1 Uniform Flow Calculations	7-26
7.6.2 Non-Uniform Flow Analysis.....	7-26
7.7 CHANNEL CONTROLS AND TRANSITIONS	7-29
7.7.1 Concept of Channel Control.....	7-29
7.7.2 Weirs, Spillways and Drop Structures	7-29
7.7.3 Hydraulic Jump.....	7-33
7.7.4 Expansions and Contractions	7-35
7.7.5 Channel Bend.....	7-37
7.7.6 Culverts	7-39
7.7.7 Bridge Piers.....	7-41
7.8 UNSTEADY FLOW	7-42
7.8.1 Unsteady Water Surface Profile Computation.....	7-42

LIST OF FIGURES

Figure 7-1(a). A natural channel cross-section – Permanente Creek near Amphitheater. This section of Permanente Creek is in the tidal reach, with a bottom elevation near the mean sea level. Hence, we have a pseudo base-flow condition. The bank-full channel corresponds well to the 1.5-year flow rate, 240 cfs. The 100-year (2400 cfs) flood plains cover the trails on both sides of the creek.	7-2
Figure 7-1(b). Guadalupe River near Blue Jay Drive. Looking upstream. 100-year flow = 12,400 cfs and 1.5-year flow = 1200 cfs. Channel has capacity to contain 100-year flow at this location. Flood plain is mostly on the left hand side with Almaden Expressway embankment on the right.	7-2
Figure 7-2(a). Permanente Creek looking downstream from Hwy 101 towards Charleston. Concrete and concrete sacks line the channel to support steep channel banks following urbanization. Construction work completed in 1963.	7-3
Figure 7-3. Channel Classification Based on Pattern and Type of Sediment Load [Schumm and Meyer 1979].....	7-5
Figure 7-4. Three-Dimensional Coordinate System.....	7-7
Figure 7-5. Definition Sketch for the Equation of Continuity.....	7-8
Figure 7-6. Application of Bernoulli Equation to Open Channel Flows.....	7-11
Figure 7-7. Specific Energy Applied to Channel Bottom Elevation Change	7-13
Figure 7-8. Specific Energy Applied to Channel Width Change.....	7-13
Figure 7-9. Changes in Flow Regime Produced by Increase in Channel Bottom Elevation ..	7-17
Figure 7-10. Definition Sketch for Application of the Momentum Equation	7-18
Figure 7-11. Application of the Momentum Function	7-20
Figure 7-12. Definition Sketch for the Resistance Equation.....	7-21
Figure 7-13. Relationship Between Absolute Boundary Roughness and the Manning's n	7-24
Figure 7-14. Manning's n in grassed channels.....	7-25
Figure 7-15. Critical flow at a change of slope.....	7-27
Figure 7-16. Longitudinal Flow Profiles on a Mild Slope	7-28
Figure 7-17. Longitudinal Profiles on Various Types of Slopes.....	7-29
Figure 7-18. Sharp-Crested Weir	7-30
Figure 7-19. Typical Designs of Rectangular and Triangular Weirs.....	7-31
Figure 7-20. Broad-Crested Weirs.....	7-32
Figure 7-21. Flow Over a Drop Structure.....	7-33
Figure 7-22. Hydraulic Jump at an Abrupt Rise, After J. W. Forster and R. A. Skrinde [1950].....	7-34
Figure 7-23. Hydraulic jump at an abrupt drop, after E. Y. Hsu [1950].....	7-35
Figure 7-24. Sudden Expansion.....	7-36
Figure 7-25. Transverse Bed Profiles Required at a Channel Bend for Constant q	7-38
Figure 7-26. Schematic Wave Pattern at a Circular Curve	7-39

LIST OF TABLES

Table 7-1 Classification of Grass by Length and Stand.....	7-25
Table 7-2 Transition Loss Coefficient.....	7-37
Table 7-3 Short and Steep Box Culvert Discharge Relationships.....	7-40
Table 7-4 Culvert Entrance Loss Coefficients	7-41

BIBLIOGRAPHY

CHAPTER 7. BASIC PRINCIPLES

7.1 OPEN CHANNEL AND ITS PROPERTIES

An open channel is a conduit in which water flows with a free surface exposed to the atmosphere. The free surface – an added dimension of freedom – renders treatment of fluid motion in the channel considerably more difficult than that of a closed conduit, and distinguishes the open channel flow as a subject of study by itself.

7.1.1 Types of Open Channel

Natural and Artificial Channels

An open channel may be classified in many different ways depending on the purpose of the classification. As are described in the Introduction section, we may divide the creeks in the Santa Clara County into 530 miles of *natural* channels and 150 miles of *artificial* channels. *Natural* channels refer to those waterways having a naturally formed physical boundary made up of natural materials such as soil, gravel, rocks and vegetation. These natural materials are likely to move or be transported when water flows, resulting in a channel geometry that may change with time. Many researchers have endeavored to examine the various forms of channel geometry and categorize them into stages in an attempt to rationalize natural channel evolution. Examples of this school of thoughts include Lane [1957], Leopold and Wolman [1957], Schumm [1963], and Rosgen [1996].

When allowed to develop in a natural state, most channels will exhibit a cross-section that consists of a base-flow area, a bankfull channel, and flood plains. In the Santa Clara Valley, natural base flows seldom exist, because our climate only supports intermittent flows during the winter season, and for those creeks that have reservoir release or pumped flow, the flow rate is usually too small to move sediment and shape the channel. The bankfull flow is defined by Dunne and Leopold [1978] as the discharge that is most effectively moving sediment, and generally doing work that results in the average morphologic characteristics of the channel. Flood plains are the areas above the bank-full level, but contained within the river valley. Examples of this *natural* channel are illustrated in Figure 7-1.

When humans establish homestead in the river valley, we install artificial materials in the channel to prevent it from further evolving and encroaching into our properties or causing floods. Thus result the *artificial* channels. Artificial channels typically consist of a physical boundary of hard materials, often referred to as hardscape, such as concrete, rocks, concrete sacks, gabions, articulated concrete blocks, etc. The hardscape is meant to stop the channel geometry from moving. But often fails to do so because we neglect to address the morphologic process that causes channel evolution. Chapters 8 and 9 will discuss in more details of the fluvial geomorphologic analysis. Typical *artificial* channels are shown in Figure 7-2.



Figure 7-1(a). A natural channel cross-section – Permanente Creek near Amphitheater. This section of Permanente Creek is in the tidal reach, with a bottom elevation near the mean sea level. Hence, we have a pseudo base-flow condition. The bank-full channel corresponds well to the 1.5-year flow rate, 240 cfs. The 100-year (2400 cfs) flood plains cover the trails on both sides of the creek.



Figure 7-1(b). Guadalupe River near Blue Jay Drive. Looking upstream. 100-year flow = 12,400 cfs and 1.5-year flow = 1200 cfs. Channel has capacity to contain 100-year flow at this location. Flood plain is mostly on the left hand side with Almaden Expressway embankment on the right.



Figure 1-2(a). Permanente Creek looking downstream from Hwy 101 towards Charleston. Concrete and concrete sacks line the channel to support steep channel banks following urbanization. Construction work completed in 1963.



Figure 7-2(b). Permanente Creek looking upstream at Hwy 101. Concreted steep banks were constructed in early 1960's to support urban development.

Straight, Meandering and Braided Channels

The natural channel evolution process is a result of water seeking a balance between its kinetic energy and the resistive energy. The process depends on flow rate, sediment load, sediment materials, channel vegetation, and channel geometry. At different stages of the evolution, the channel may exhibit different configurations. These are referred to as planforms. The basic channel planforms include straight, meandering and braided. The classification is based on the sinuosity of the channel, defined as the thalweg length divided by the down-valley length. Schumm and Meyer [1979], based on observations of the Great Plains rivers, developed trends associating planform to sediment load, as depicted in Figure 7-3 which was excerpted from US Army Corps of Engineers [1994]. Mollard and Janes [1984] presented a more extended set of classifications of planforms with associated environmental conditions. It suffices to say that a natural channel may develop into different planforms depending on its flow and sediment characteristics. In urbanized areas human interventions often introduce hardscape into the channel and change the natural morphologic process and make planform classification difficult. This fact should be recognized when working with urbanized streams.

Perennial, Intermittent and Ephemeral Channels

In Santa Clara Valley, most creeks dry up naturally in the summer in recent years. Classifying based on the water-flowing period, channels may be divided into perennial, intermittent and ephemeral types [North Carolina Urban Water Consortium 2001]. Perennial is defined as water flowing more than 90% of the time; intermittent is flowing during wet periods, 30 to 90% of the time and in a continuous and well-defined channel; and ephemeral is flowing only during storms and may or may not have a well-defined channel such as those in the desert areas of the southwestern US.

Before the 1950's, prior to excessive groundwater pumping caused significant valley floor subsidence, most creeks in the Santa Clara Valley flew year-round. Today, some of the creeks have reservoir release or artificial recharge and are flowing year-round, such as Coyote Creek downstream from Anderson Reservoir, Los Gatos Creek downstream from Lexington Reservoir and Guadalupe River. Most other creeks in this Valley are intermittent and flow 5 to 8 months of a year. Ephemeral creeks typically occur in arid and desert environments, and do not exist locally in the Santa Clara Valley. Public agencies sometimes associate this classification with definition of requirements of riparian buffer zones.

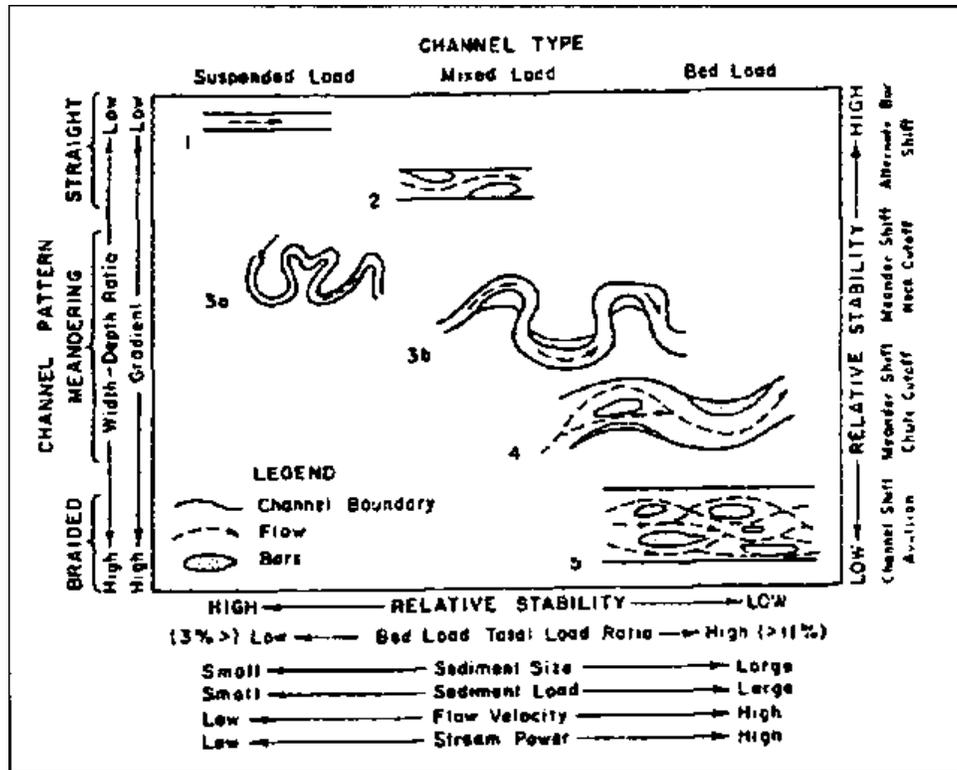


Figure 7-3. Channel Classification Based on Pattern and Type of Sediment Load [Schumm and Meyer 1979]

7.1.2 Types of Flow

Independent of the channel types, open channel flows may be classified according to flow characteristics. Each type of flow needs to be analyzed using methods that are appropriate for the flow characteristics. These types include:

Steady and Unsteady Flows

The steadiness of flow is a reference with respect to time. Steady flow means the flow characteristics, such as velocity and water depth, remain constant over the time period in discussion. Random turbulent fluctuations are not considered here, because the time scale for steady flow is large compared to that of turbulent fluctuations. In mathematical terms, for steady flow, all differentiations with respect to time drop out, i.e.,

$$\frac{\partial V}{\partial t} = 0$$

$$\frac{\partial Y}{\partial t} = 0$$

where V = average velocity, Y = water depth in the channel, and t = time. In nature, steady flow almost never occurs in the Santa Clara Valley.

For unsteady flow, on the contrary, time remains as a variable and differentiations with respect to time remain in the governing equations.

Uniform and Non-Uniform Flows

Flow uniformity is a reference with respect to space. When the flow characteristics, such as velocity and depth, remain constant along the channel, the flow is described as a uniform flow in the longitudinal direction. Hence,

$$\frac{\partial V}{\partial x} = 0$$

and

$$\frac{\partial Y}{\partial x} = 0$$

where x = coordinate in the longitudinal direction. In reality, because of the relatively small sizes of our watersheds and the continuous change in channel gradient from the Coast Range or Diablo Mountains to the Bay, creek flow rarely remains constant from location to location. Hence, channel velocity and water depth change as the flow goes downstream. These flows are called nonuniform flows, or spatially varied flows. More descriptions of these flows will be provided later in this chapter.

Turbulent and Laminar Flows

In the absence of velocity variation in any direction, the fluid in a channel will move in layers, or laminae, whence the expression "laminar flow." All fluids have their inherent viscosity. Laminar flow happens when the fluid moves in a viscous state, or when the inertia effect is relatively insignificant. The inertia of a flow may be defined by the product of density and convective acceleration, hence proportional to $\rho v^2 / R$, where v is the velocity and R the hydraulic radius.

The viscous effect will be proportional to $\mu v / R^2$ where μ is the dynamic viscosity. The ratio of these two quantities gives a measure of the relative significance of inertial effect over viscous effect.

$$\frac{\rho v^2 / R}{\mu v / R^2} = \frac{vR}{\nu} = \text{Reynolds Number} = \mathbf{Re}$$

This is the definition of Reynolds number, where ν is the kinematic viscosity. It has been found that when Re has a magnitude of the order of 1000, the flow is in a laminar state. Higher than

this number, the flow transitions into a state of intermittent turbulence, and even higher the flow becomes fully turbulent.

In our creek environment, we nearly always have turbulent flows. Natural water at our annual average temperature of 70°F has a kinematic viscosity of approximately 10^{-5} ft²/sec. Hence, it will take a value of 10^{-2} of the product of velocity and hydraulic radius (or depth) to bring the Reynolds number to 1000. This seldom occurs. Therefore, in our creeks we need to be concerned with turbulent flows only.

After describing the basic properties of the open channel flows, we are ready to use mathematical tools to derive equations that will help us describe and understand the creeks.

7.2 DIMENSIONALITY

In natural creeks, the flow is truly 3-dimensional, meaning flow characteristics change in all 3 physical directions, in addition to change in time. Mathematically, the 3 dimensions of a water body are customarily referred to as the longitudinal (x), transverse (y) and vertical (z) directions. The independent variables are the flow characteristics, namely the flow rate, sediment discharge, velocity, water depth, etc. They may be functions of all or part of the dependant variables, including channel geometry (width, depth and shape), channel roughness, channel slope, specific gravity of water, sediment particle characteristics (size, shape & specific gravity), etc.

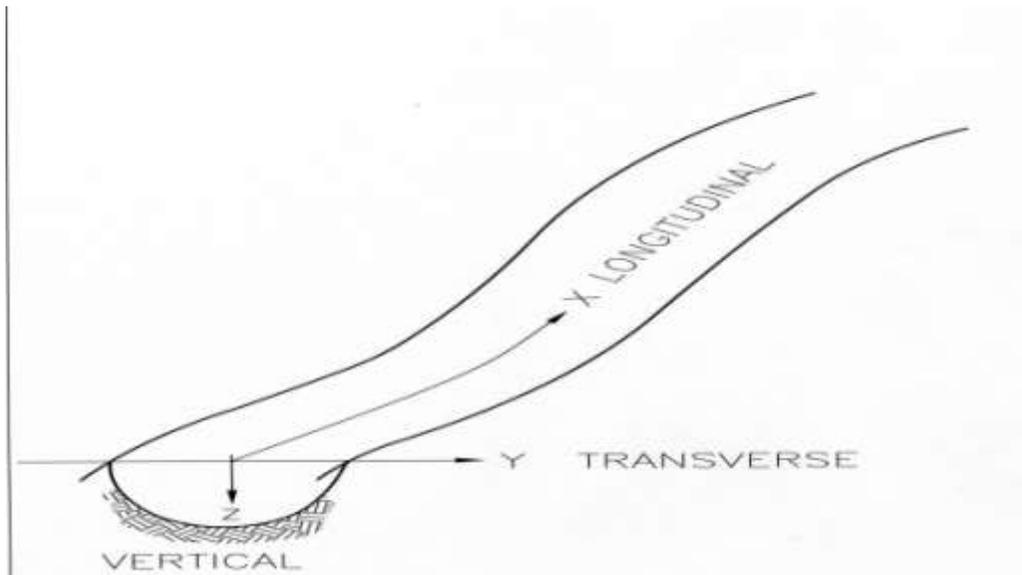


Figure 7-4. Three-Dimensional Coordinate System

When solving open channel problems analytically, we often attempt to simplify the problem by making approximations. One basic approximation is to average the characteristics in a particular direction, and hence, remove the dependency on that spatial dimension. An example is when dealing with a long and relatively narrow reservoir, such as the Lexington Reservoir, it seems reasonable to assume that changes in the transverse direction may be either neglected or averaged. Hence, the reservoir characteristics, such as temperature and velocity distributions, are only 2-dimensional. This significantly simplifies the problem and treatment.

The 2-dimensional computer program for hydrodynamic and water quality simulation in reservoirs, CE-QUAL-W2 [USACE 2003], fits this category of application.

Another example of making such approximations is when the purpose of the analysis is met without having to deal with the 3-dimensional variations in the water body. When conducting the flood control projects, one of the major objectives is to compute the peak flood water levels at various locations of the creek. In most cases, it is not necessary to find out how velocities change from the right to left bank or from the top to bottom. Hence, the change in transverse and vertical directions may be omitted through an averaging process. That is the basic approximation adopted in the current version of HEC-RAS [USACE 2006], the 1-dimensional program to compute water surface profiles in rivers.

7.3 CONSERVATION OF MASS

In many open channel problems, it helps to examine the basic equations of conservation of mass, momentum and energy to understand a flow situation and be able to analyze it. These basic equations are briefly discussed in the following.

Consider the open-channel flow in Figure 7-5. Assume the fluid is incompressible, as is for all of the natural conditions we encounter. Take a control volume bounded by Sections 1 and 2, the impermeable bottom and the free surface.

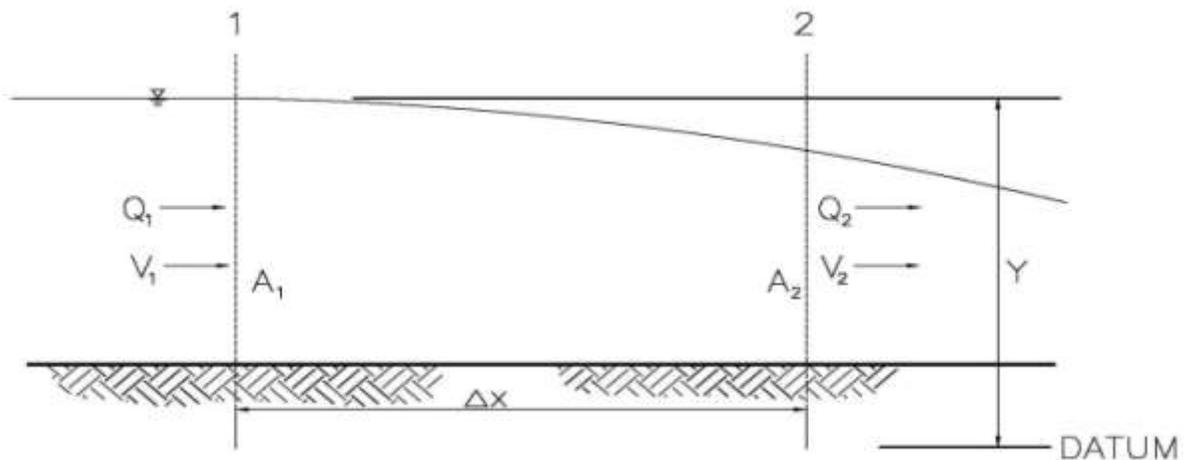


Figure 7-5. Definition Sketch for the Equation of Continuity

The net rate of flow out of a control volume is

$$Q_2 - Q_1 = \frac{\partial Q}{\partial x} \Delta x$$

The partial derivative is necessary because Q may be changing with time as well as with the distance along the channel. Now if Y is the height of the water surface above datum and B is the channel width at the water surface, then the control volume is increasing at the rate

$$B \frac{\partial Y}{\partial t} \Delta x$$

The law of conservation of mass states that the rate of flow out of the control volume must be equal in magnitude but opposite in sign to the rate of increase of the control volume, i.e.,

$$\frac{\partial Q}{\partial x} = -B \frac{\partial Y}{\partial t} \quad \text{or} \quad \frac{\partial Q}{\partial x} + B \frac{\partial Y}{\partial t} = 0 \quad (7-1)$$

This is the equation of continuity for unsteady open channel flow. For a case of steady flow,

$$\frac{\partial Y}{\partial t} = 0$$

and the equation becomes

$$\frac{\partial Q}{\partial x} = 0$$

which follows that

$$Q_1 = V_1 A_1 = Q_2 = V_2 A_2 \quad (7-2)$$

which is the equation of continuity for steady flow of an incompressible fluid. In this case, if the channel geometry changes, A_1 and A_2 may be different, and V_1 and V_2 will differ accordingly, but Q_1 and Q_2 must equal to fulfill the requirement of steady flow.

7.4 CONSERVATION OF ENERGY

Consider a single fluid element moving in an arbitrary direction s . The conservation of energy equation may be derived from Newton's second law of motion,

$$F = ma$$

where F is the resultant force acting on the element, m the mass of the element and a the acceleration. The forces driving the element include the pressure head and the weight of the fluid. If ρ is the mass density of the fluid, p the pressure force the element experiences, y the elevation of this element above datum, and a_s its acceleration in the s direction, the law of motion becomes,

$$\frac{\partial}{\partial s}(p + \gamma y) + \rho a_s = 0 \quad (7-3)$$

This is the Euler equation. The acceleration may be written as a derivative

$$a_s = \frac{dv}{dt} = v \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}$$

Substituting into Equation (7-3), we have the equation for unsteady flow, such as a flood wave moving down a creek. For the time being, let's focus on steady flow, in which case Eq. (7-3) becomes

$$\frac{\partial}{\partial s}(p + \gamma y) + \rho v \frac{\partial v}{\partial s} = 0$$

Integrating this equation we obtain

$$p + \gamma y + \frac{1}{2} \rho v^2 = \text{constant}$$

or

$$\frac{p}{\gamma} + y + \frac{v^2}{2g} = \text{constant, H} \quad (7-4)$$

These are alternative forms of the well-known Bernoulli equation, attributed to the Swiss mathematician Daniel Bernoulli for his introduction of the concept of "head." Since it was obtained by integrating a force equation with respect to distance, it is an energy equation. The energy equation always holds true, provided proper allowance is made for energy losses, or more properly, the dissipation of kinetic energy into heat energy.

7.4.1. Hydrostatic Pressure Distribution

In the Euler Equation (7-3), if $a_s = 0$, i.e., if there is negligible acceleration in the direction s , then

$$\frac{\partial}{\partial s}(p + \gamma y) = 0$$

Hence, $(p + \gamma y)$ remains a constant along the direction s . Using Figure 7-6(a) as an example, if s is the vertical direction along line OAB, when there is no vertical acceleration, the term $(p + \gamma y)$ becomes a constant. It is the same at points O, A and B, as the height of the water surface above datum. The pressure distribution in this case is hydrostatic, i.e., pressure is linearly proportional to water depth. Under such condition, the free surface becomes the hydraulic grade line for the flow domain.

Not all cases of open channel flow have hydrostatic pressure distribution. Take the case shown in Figure 7-6(b) for instance, where water is falling off a vertical drop structure. At the edge of

the drop structure, the flow is changing direction from horizontal to nearly vertical. Hence, along a vertical cross-section at the edge one would expect the change in velocity to be significant and the acceleration term in Eq. (7-3) not negligible. The slashed area in Figure 7-6(b) shows schematically the hydrostatic and the actual pressure distributions at this location.

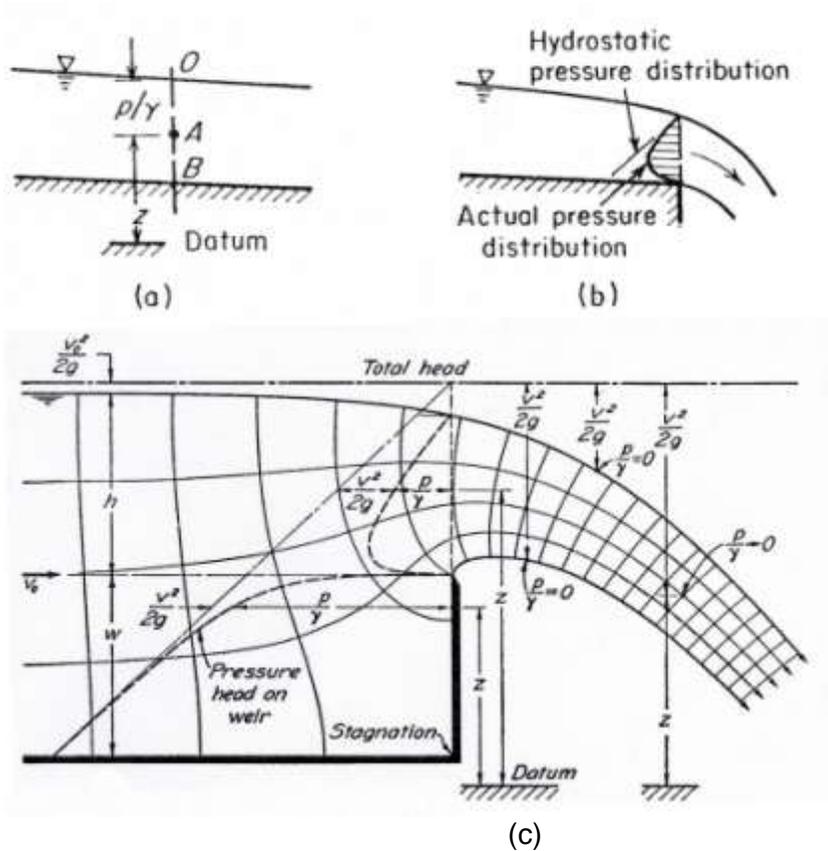


Figure 7-6. Application of Bernoulli Equation to Open Channel Flows

Figure 7-6(c), taken from p. 93 of [Rouse 1946], is a classic example of non-hydrostatic pressure distribution at the edge of a sharp-crested overflow weir. A flow net was drawn through the flow domain to scale and the different terms of the energy equation were illustrated graphically to reflect their relative significance. This is an example to show how informative an engineering sketch can be when drawn to scale and using the proper tools.

In most cases of natural creeks in our environment, the bed slope is not significant and the vertical distance may be taken as the water depth at right angles to the bed. In these cases, the free surface coincides with the hydraulic grade line, the pressure distribution is hydrostatic, the vertical curvatures and accelerations are negligible, and the Equation (7-4) holds true. Just remember that this statement breaks down in case of spillways, where the slope is significant and the pressure distribution no longer hydrostatic.

7.4.2 Specific Energy

We should introduce the concept of specific energy now. It is such a simple concept, yet it holds the key to many a complex open channel flow phenomenon, as you will see explained in the following.

Introduced in 1912 by Boris A. Bakhmeteff in Russian language, Specific Energy is defined as the energy per unit weight of water in a channel with respect to the channel bottom. Hence, for our creeks of mild slopes (hydrostatic pressure distribution), it is

$$E = y + \frac{v^2}{2g} \quad (7-5)$$

where y is the water depth and v the mean velocity. Substituting v for $\frac{Q}{A}$, where Q is the flow rate and A the flow area, we have

$$E = y + \frac{Q^2}{2g A^2} \quad (7-6)$$

In our daily practice of hydraulic-engineering analysis or design, it often helps to understand how an open channel flow will react when the channel widens or narrows, or through a step change in channel bottom, or after the confluence of a storm drain inflow. Knowing in theory how the flow will react will help us review the analysis result, catch errors in modeling, and optimize the design. The concept of specific energy allows these to happen.

Two characteristics of the concept one notices immediately. The first is that it is referring to a total energy based on the channel bottom elevation. If the channel bottom elevation rises, it will reduce the specific energy since the total energy stays the same. The second is that it is a relationship between E and y for a fixed discharge Q . For different flow rates, we will have different $E(y)$ relationships. Hence, we should plot the relationship in an (E, y) coordinate system for various Q 's to examine the trends.

For simplicity, let's assume a rectangular channel. The channel width is B and $A = By$. Let's also use a unit discharge, the discharge per unit width, represented by q in our consideration. With $q = Q/B$, Equation (7-6) becomes

$$E = y + \frac{q^2}{2g y^2}$$

and

$$(E - y) y^2 = \frac{q^2}{2g} = \text{constant} \quad (7-7)$$

This relationship has 2 asymptotes, $(E - y) = 0$ and $y = 0$. Plotted on a $E - y$ plane, the first asymptote is a 45° line of $E = y$ and the second is the E axis. The actual plot and application of this relationship is illustrated in 2 examples shown in Figures 7-7 and 7-8.

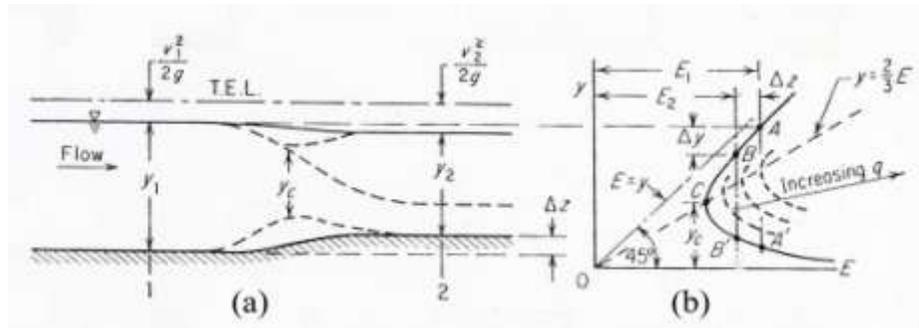


Figure 7-7. Specific Energy Applied to Channel Bottom Elevation Change

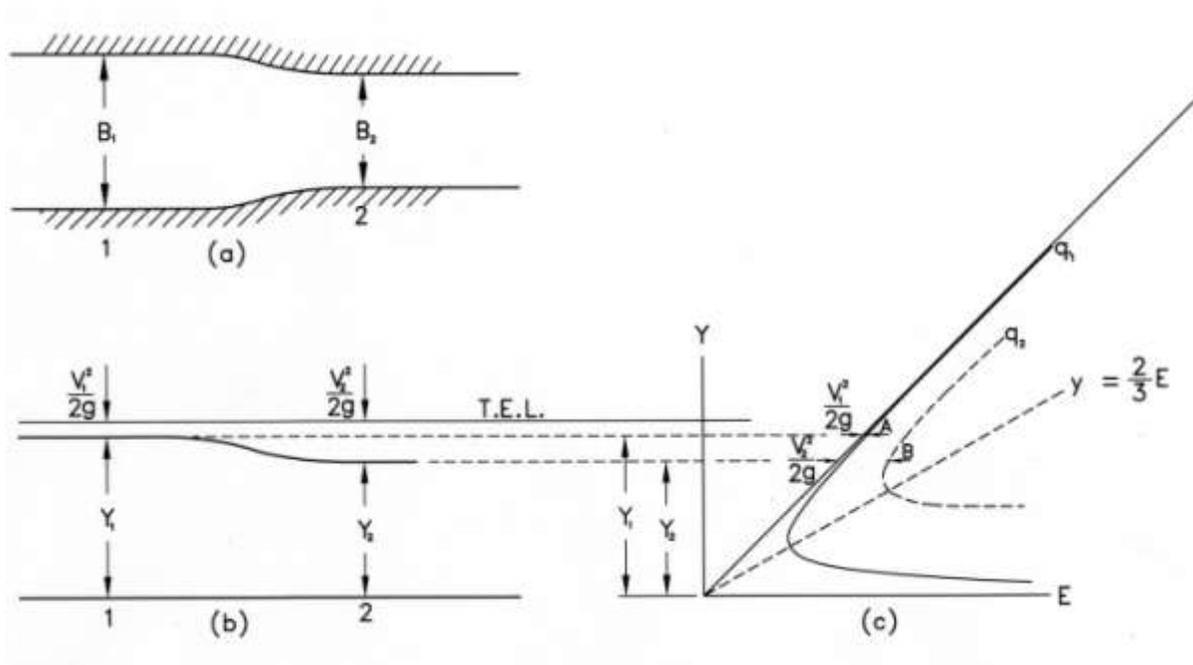


Figure 7-8. Specific Energy Applied to Channel Width Change

As shown in Figure 7-7(b), the $E - y$ relationship for a fixed q is a curve in the lower half of the 1st quadrant of the $E - y$ plane, with both ends of the curve asymptotically approaching the 45° line ($E = y$) and the E axis. For a constant y , when q increases, the specific energy E increases. Hence, the curve moves to the right and inside of those having lower values of q .

Figure 7-7(a) shows a channel having a smooth step rise on the bottom elevation. Section 1, upstream from the step rise, has a water depth y_1 and a specific energy E_1 , and is represented in Figure 7-7(b) as the point A. Since the step change occurs smoothly, we may assume that the energy loss through the step is negligible. Hence, at Section 2, downstream from the step rise, the total energy remains the same as the upstream, while the specific energy E_2 is reduced by the amount of the step rise Δz . This E_2 value corresponds to the point B on Figure 7-7(b). The y coordinate of the point B determines the water depth y_2 .

On Figure 7-7(b), each specific energy E value intersects the E - y curve at two points with two distinctive water depths, such as A and A'. This is because there are 2 real solutions to the cubic Equation (7-7), with the third solution in the negative y domain, hence, physically improbable. The 2 real solutions represent the 2 possible water depths for each specific energy E and flow rate q . The points on the upper limb of the curve represent a regime of flow that is deeper in depth and slower in velocity. The lower limb on the other hand is shallow and fast. Both limbs meet at the crest of the curve, point C. The physical importance of these regimes will be clear in the next section. Suffice to say at this time, for a point on the upper limb, a small step rise in bottom elevation will reduce the water depth. Had the point originally been on the lower limb, the same step rise will actually increase the water depth.

Let us examine Figure 7-8 now. It is a channel with its width reduced through a smooth transition. At section 1, the width is B_1 and depth y_1 . This flow condition is represented on Figure 7-8(c) as the point A. As the channel width reduces gradually to B_2 , assuming the energy loss is negligible, the specific energy remains the same, but the discharge per unit width increases to q_2 , and the flow condition is represented by the point B. It is seen that the resulting water depth y_2 will be lower than y_1 and the velocity will be faster.

Hence, the specific energy is a useful tool to analyze open channel reactions to changes in channel characteristics. More of its usefulness will be covered in subsequent sections discussing critical flow and super- and sub-critical flow regimes. The concept of unit discharge also helps, because in many of our creeks in the Santa Clara Valley, the channels may be approximated into rectangular cross-sections for an engineering examination.

7.4.3 Critical Flow and the Froude Number

We mentioned that the crest of the E - y curve, point C on Figure 7-7(c), separates 2 flow regimes. What other physical significance does this point C have? This obviously is a point of minimum specific energy. All other flow conditions for the same Q , or q , require higher energy. Mathematically, this is the point where $dE/dy = 0$. In the literature you have seen this point identified as the critical flow condition. To examine this condition, let us go back to Eq. (7-6). Differentiating Eq. (7-6) with respect to y , keeping Q as a constant, we have

$$\frac{dE}{dy} = 1 - \frac{Q^2}{g A^3} \frac{dA}{dy} \quad (7-8)$$

Using a rectangular channel as an example again, we have $dA/dy = B$, and Eq. (7-8) becomes

$$\frac{dE}{dy} = 1 - B \frac{V^2}{gA} = 1 - \frac{V^2}{gy}$$

Hence, at point C where $dE/dy = 0$, we have

$$\frac{y_c}{2} = \frac{V_c^2}{2g} \quad (7-9)$$

The subscript c denotes the critical flow condition. Since

$$E_c = y_c + \frac{V_c^2}{2g}$$

we have $E_c = \frac{3}{2} y_c$ or $y_c = \frac{2}{3} E_c$ (7-10)

Also, since $q_c = V_c y_c$, Equation (7-9) yields

$$y_c = \sqrt[3]{\frac{q_c^2}{g}} \quad (7-11)$$

All Equations (7-9) through (7-11) depict y_c in forms that may be easily calculated from the specific energy, velocity or unit discharge. They also show that the critical water depth increases with unit discharge, and is equal to 2/3 of the specific energy, as represented by the dashed line in Figures 7-7(b) and 7-8(c) that connects the origin and the crests of the E-y curves.

Equation (7-9) also has several other important physical meanings. In the 19th century, when an English engineer named Froude did his model tests on resistance on ships, he developed a similitude relationship to evaluate relative effects of inertial and gravitational forces. This relationship became the famous Froude Number Fr . Used in our rectangular channel the relationship is

$$Fr = \frac{v}{\sqrt{gy}}$$

We see that at critical flow, $v = \sqrt{gy}$, the Froude Number becomes 1. It also follows that Froude numbers less than unity indicate flow at depths greater than the critical depth and at velocities less than the critical velocity; and Froude numbers greater than unity indicate flow at depths less than the critical depth and at velocities greater than the critical velocity. The former is referenced in some literature as the tranquil flow and the latter rapid flow. This distinction in flow behavior also demonstrates that the configuration of the free surface should be a unique function of the Froude Number of the approaching flow. That is the basis for using the Froude similitude to conduct scale-model studies.

Another observation is that \sqrt{gy} is the velocity with which a long or low-amplitude wave propagates in water of depth y , i.e., the wave celerity. When one throws a stone into a shallow flowing stream, the ripples (small gravity waves) the stone generates will travel at the approximate speed of \sqrt{gy} . If the flow velocity is higher than the wave celerity, to a stationary observer on the bank the ripples will seem to only propagate in the downstream direction. This is a crude but effective way to determine flow regime in the field.

7.4.4 Subcritical and Supercritical Flows

We have established that at critical flow the water is moving just as fast as the wave resulting from a small disturbance will move relative to the water. When the flow velocity is faster than critical, the depth of flow will be less than that of the critical depth, and this flow regime, represented by the lower limb of the curves on Figure 7-7, is called supercritical. When the flow velocity is slower than critical, the depth of flow will be higher than that of the critical depth, represented by the upper limb of the curves of Figure 7-7, and the flow regime is called subcritical.

For subcritical flows, the Froude number is less than 1, and for supercritical flows, the Froude number is greater than 1. For subcritical flows, as the name of tranquil flow implies, the flow is smooth and slow. The transition to other flow regimes, through channel boundary changes, is usually smooth as well. When the flow is near critical condition, the flow becomes less stable. A small disturbance in the channel, such as a rock in the creek, will generate relatively much larger, compared to the rock size, undulating waves on the surface. This in turn may cause splashing and overflow conditions. Hence, one needs to consider freeboard requirements more carefully when the flow is near or above critical.

The above is also true for supercritical flows. Supercritical flows will move at a speed faster than the waves generated by any disturbance in the channel, be it a log trapped in the channel, a bridge pier or a rough transition on the channel wall. As a result the consecutive circular rings of the waves generated by the disturbance form an oblique wave front pointing in the direction of flow. This wave front requires additional channel wall height to contain the flow. It will also propagate for a long distance before its energy is dissipated. Hence, when the supercritical flow condition is unavoidable, make sure that you consider the oblique wave and possibilities of standing waves in the channel.

7.4.5 Applications of the Energy Principle

Before we leave the subject of specific energy, you should be familiar with the application of Figures 7-7 and 7-8. A set of pictures shown in Figure 7-9 illustrates how flow transitions from one regime to another may take place. Excerpted from [Rouse 1978], these pictures demonstrate that in order to transition from subcritical to supercritical, or vice versa, the flow has to go through critical at some stage. The transition may be smooth, as in Figure 7-9(b), and

may be rough as in Figure 7-9(d). Both upstream and downstream conditions of all 4 pictures should be represented by the points A, B, B' and A' in Figure 7-7, if the slight energy loss in the last 2 pictures are ignored.

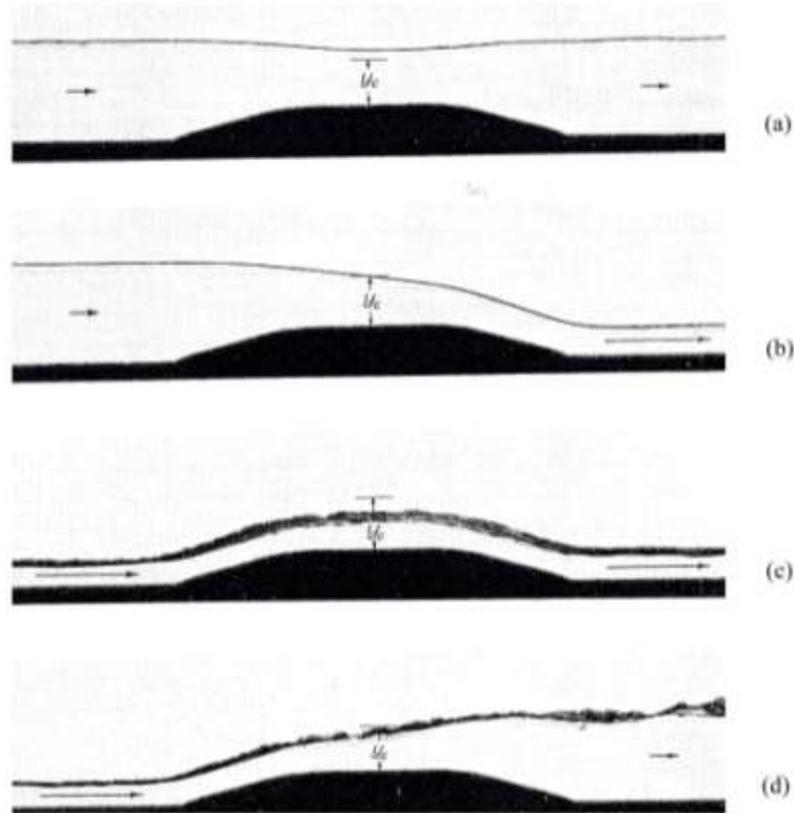


Figure 7-9. Changes in Flow Regime Produced by Increase in Channel Bottom Elevation

You should also realize that there is a limit in how much bottom elevation you can raise or channel width you can reduce, before you run into a problem of “choking.” In case of bottom raising, since the unit discharge remains the same, the flow stays on the same E-y curve. On Figure 7-7, we begin at point A or A'. As the elevation is raised, the point tracks back through B or B' toward the critical point C. When the elevation is raised more than the horizontal distance (on the E coordinate) between points A and C, the flow will drop off the curve. This simply means that the channel cannot sustain the same flow any more. Hence, the flow rate has to drop, and choking occurs. A similar phenomenon happens with width reduction. The specific energy curve is the tool to use to analyze and design channel transitions, which will be discussed some more in Section 7.7 below.

7.5 CONSERVATION OF MOMENTUM

In previous discussions of specific energy, we assumed that the head loss in the channel section was negligible, and applied the principle of conservation of energy to determine the flow characteristics. Unfortunately, there are many cases in which the head loss cannot be ignored, and the conservation of energy alone is not sufficient to solve the problem. In those cases, we apply the principle of conservation of momentum.

Integrating Newton's second law of motion over a control volume between sections 1 and 2, we have

$$F_x = \rho \int [(V_x)_2 - (V_x)_1] dQ \quad (7-12)$$

where F_x is the summation of all forces acting on the control volume in the x direction, V_x is the velocity in the x direction, and Q is the discharge. Using Figure 7-10 to represent the channel and control volume, this momentum equation becomes

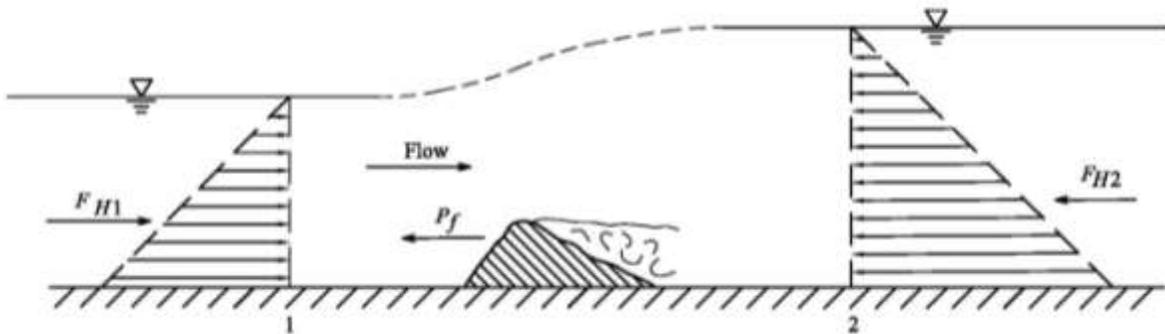


Figure 7-10. Definition Sketch for Application of the Momentum Equation

$$F_{H1} - F_{H2} - P_f = \rho Q v_2 - \rho Q v_1 \quad (7-13)$$

where F_H is the hydrostatic thrust on the flow section, and P_f is the force exerted by the obstacle on the flow. For the particular case of a rectangular channel, we consider a unit width of the channel, so that Eq. (7-13) becomes

$$\frac{\gamma y_1^2}{2} - \frac{\gamma y_2^2}{2} - P_f = q \rho v_2 - q \rho v_1 \quad (7-14)$$

where P_f is now defined as the force per unit width of the channel. This equation may be used in its present form to solve any particular problem. Note that P_f includes skin friction from the bottom and component of the weight of the water body on the bottom.

7.5.1 The Momentum Function

Making the substitution $v = \frac{q}{y}$ in Eq. (7-14) and dividing throughout by γ , we will have

$$\begin{aligned}\frac{P_f}{\gamma} &= \left(\frac{q^2}{g y_1} + \frac{y_1^2}{2} \right) - \left(\frac{q^2}{g y_2} + \frac{y_2^2}{2} \right) \\ &= M_1 - M_2\end{aligned}\tag{7-15}$$

where

$$M = \frac{q^2}{g y} + \frac{y^2}{2}\tag{7-16}$$

is called the *momentum function*. Eq. (7-16) was originally developed by [Bresse 1860] in his study of the hydraulic jump. Note that the terms in this function are all forces per unit weight of water per unit width. Hence, the function has been variously called specific force, momentum flux, or force plus momentum. It is not important what it is called, but it is very important to understand its applications, as will be explained next.

7.5.2 The M-y Relationship

There is a good reason to derive the momentum principle in the form of Eq. (7-15) and define the momentum function in the form of Eq. (7-16). Note that the form of this function is similar to that of the specific energy, Eq. (7-7). Plotted as a relationship between M and y for a fixed q, this M-y curve is similar the E-y curve, except that it has only one asymptote, $y = 0$. It has upper and lower limbs representing subcritical and supercritical flow, respectively. At the minimum value of M, where $\frac{dM}{dy} = 0$, it is easy to show that

$$\frac{y}{2} = \frac{V^2}{2g}$$

This is the criterion for the critical state of flow, derived earlier in Eq. (1-9). Hence, the crest of the M-y curve also represents the critical flow. These characteristics make the momentum function useful in combination of the specific energy relationship.

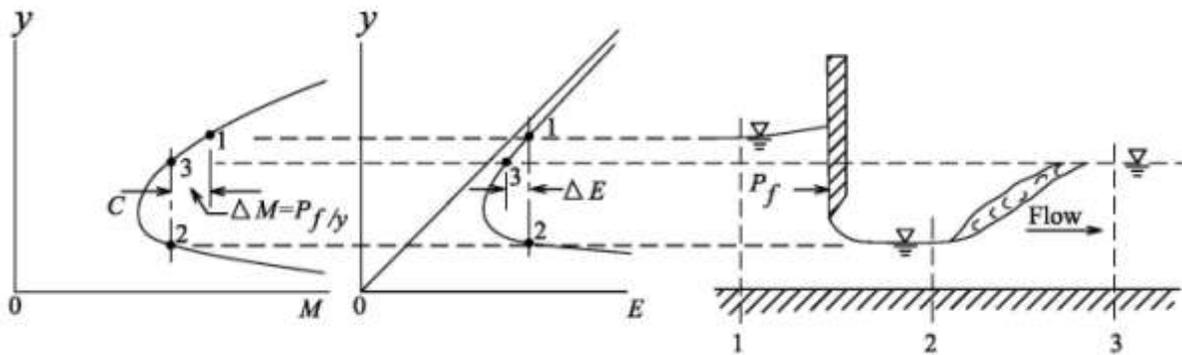


Figure 7-11. Application of the Momentum Function

Let us illustrate the application of the momentum function in the case of a hydraulic jump downstream of a sluice gate, as shown in Figure 7-11. We have designated sections 1 and 2 across the sluice gate, and sections 2 and 3 across the hydraulic jump. At these sections, it is important to know that

$$E_1 = E_2, \text{ but } M_1 \neq M_2$$

$$M_2 = M_3, \text{ but } E_2 \neq E_3$$

Starting at section 1, given the upstream water depth, we can locate point 1 on the E-y curve. A vertical line across the curve through point 1 will determine the water depth at section 2 (y_2). Locate this water depth on the M-y curve and draw a vertical line to locate point 3 after the jump. The horizontal difference between points 1 and 2 on the M-y curve provides the force acting by the sluice gate on the water. The horizontal difference between points 2 and 3 on the E-y curve provides the energy loss through the hydraulic jump.

The example brings up a couple points of observation. One is that if the downstream control in the channel provides a water depth at section 3 different from that is shown, the hydraulic jump will not happen. Given a y_2 , only one y_3 on the M-y curve exists. These 2 depths are called the conjugate or sequent depths of a hydraulic jump and will be discussed in Section 7.7.3. If the tail water depth is lower than y_3 , the flow will go through a transition that will be discussed in Section 7.6.2 for nonuniform flow analysis. If the tail water depth is higher than y_3 , the jump may be partially or completely drowned out.

Another point of observation is that if the opening under the sluice gate is small, there will be head losses, and $E_1 = E_2$ becomes only an approximation. Also note that immediately downstream from the sluice gate, the water surface is curved. Flow exiting the sluice gate with a significant vertical velocity component, and the pressure distribution along the bottom is not hydrostatic. Actually, the pressure will be higher than hydrostatic just downstream from the sluice gate. Hence, section 2 has to be located far enough downstream to allow the conservation principles that we derived to be applicable.

7.6 FLOW RESISTANCE

The conservation principles that we discussed set the foundation for hydraulic analysis. To carry it one step further, we need a means to estimate the losses that will occur for various channel geometries and flow conditions. A significant component of the head losses occurs due to the resistance at the boundary of the channel. To examine the boundary resistance, let us analyze a flow element of the length Δx shown in Figure 7-12.

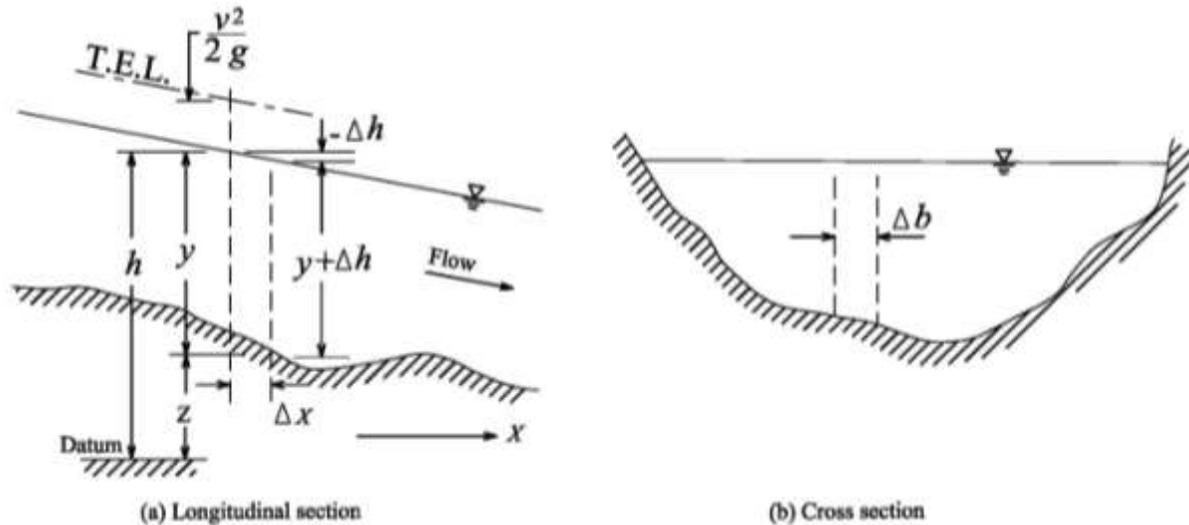


Figure 7-12. Definition Sketch for the Resistance Equation

Assuming the channel slope is small and the pressure distribution hydrostatic, the total horizontal hydrostatic thrust on the element is $-\gamma A \Delta h$, where A is the cross-sectional area, and the force is taking positive in the downstream direction. This force is resisted by a shear force along the boundary of the magnitude $-\tau_o P \Delta x$, where τ_o is the bottom shear stress.

Now, consider the state of *uniform flow*, where there is no net acceleration, and the 2 forces have to balance. Hence,

$$\tau_o = -\gamma \frac{A \Delta h}{P \Delta x}$$

Since $A/P = R$ and is called the hydraulic radius, and $-\Delta h/\Delta x = S_o$, the bed slope, the bottom shear stress may be expressed by

$$\tau_o = \gamma R S_o \quad (7-17)$$

Now, consider a more general case of *non-uniform flow*, where the net force is not zero. If we only consider a *steady state* case, where the partial derivative of velocity with respect to time vanishes and the only acceleration term is convective, we have

$$-\gamma A \Delta h - \tau_o P \Delta x = \rho A v \frac{dv}{dx} \Delta x$$

Rearranging,

$$\begin{aligned} \tau_o &= -\gamma R \left(\frac{dh}{dx} + \frac{v}{g} \frac{dv}{dx} \right) \\ &= -\gamma R \frac{d}{dx} \left(h + \frac{v^2}{2g} \right) \\ &= \gamma R S_f \end{aligned} \quad (7-18)$$

where S_f is the slope of the total energy line, sometimes called the energy slope or friction slope. Hence, we see that for any steady state flow the shear stress can be written as

$$\tau_o = \gamma R S \quad (7-19)$$

provided that the slope S is properly defined. In 1768 a French engineer named Chezy was given the task of designing a canal for the Paris water supply. He came up with a formula

$$v = C \sqrt{RS} \quad (7-20)$$

known as the Chezy equation. It is seen that the Chezy equation essentially is the same as Eq. (7-19), if you replace τ_o by what a dimensional analysis will provide

$$\tau_o = a \rho v^2$$

where a is a dimensionless coefficient which may depend on the boundary roughness, cross-sectional geometry and Reynolds number. It is noted that the Chezy equation is also applicable to any steady state condition, provided the slope is properly defined.

Nikuradse, Colebrook and White [ASCE 1963] in the 1930's conducted extensive experiments on roughness factors and friction losses for pipe flows. A part of the results came to become the Darcy-Weisbach equation for head losses in pipes

$$h_f = f \frac{L}{D} \frac{v^2}{2g} \quad (7-21)$$

where h_f is the head loss, f the Darcy-Weisbach friction factor, D the hydraulic radius and L the length of pipe. It is seen that the Chezy Eq. (7-20) and Darcy-Weisbach Eq. (7-21) have the same form as well. There are linked by the relationship

$$C = \sqrt{\frac{8g}{f}} \quad (7-22)$$

In the late 19th and early 20th century, a relationship similar to that of the Chezy equation was developed. This became known as the *Manning's Equation*. In English units, the equation is

$$v = \frac{1.49 R^{2/3} S^{1/2}}{n} \quad (7-23)$$

where v is in feet per second and R in feet. In metric units, the coefficient 1.49 disappears and the length unit is the meter. You will see in some references the coefficient in Manning's Equation (7-23) of 1.486 instead of 1.49. That is really not necessary. Given the nature of the formula and accuracy of the data, a coefficient of 1.49, or even 1.5, should suffice.

When originally developed, the Manning's n is a characteristic only of the boundary material. It has a dimension of length to the 1/6 power ($L^{1/6}$). It can be shown that

$$n = 0.031 d^{1/6} \quad (7-24)$$

where d is the particle size on the channel bed in feet. However, in actual applications, it is difficult to use this relationship, unless the channel is of a regular shape and homogeneous materials. As will be found in references described later, engineers have included variations in channel geometry and state of meandering in the composition of n so that it became a more empirical value.

Comparing the Chezy's Equation (7-20) with Manning's Eq. (7-23), we see that

$$C = 1.49 \frac{R^{1/6}}{n} \quad (7-25)$$

Then from Eq. (7-22) and (7-25), we have

$$\frac{1}{\sqrt{f}} = \frac{1.49}{\sqrt{8g}} \frac{R^{1/6}}{n} \quad (7-26)$$

Since there is a significant amount of data to relate the Darcy-Weisbach f to the relative roughness factor R/k , where k is the absolute roughness of the surface, and we know that n is proportional to the absolute roughness of the channel boundary, we can examine now how the

dimensionless variables $R^{1/6}/n$ varies with R/k . Plotted in Figure 7-13, we see that a three-fold increase in n corresponds roughly to a thousand-fold increase in the absolute roughness k .

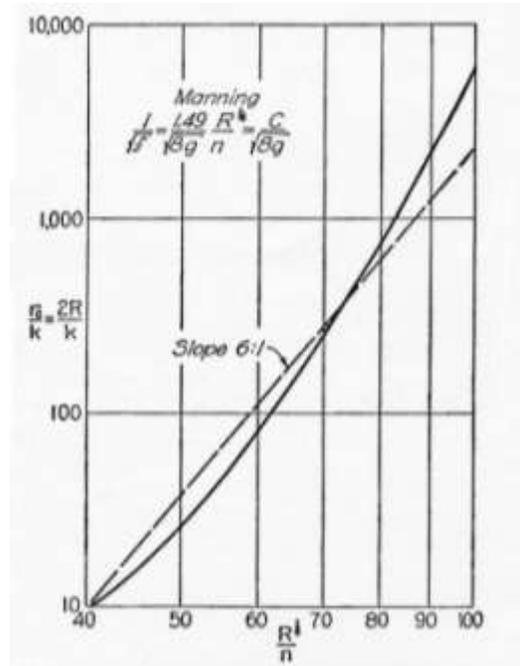


Figure 7-13. Relationship Between Absolute Boundary Roughness and the Manning's n

We also know from the more rigorously conducted experiments in pipes, that the relationship between the 2 parameters on this figure should follow a straight line with a slope of 6:1. The deviation of the Manning's equation to this line represents the error, or complications, in this equation. It is seen that the Manning's equation is most dependable for moderate values of relative roughness. Also, since the Manning's Equation does not include the effect of viscosity, one may expect the equation to be least dependable at low values of the Reynolds number.

Chow [1959] provides a series of photographs showing calibrated Manning's coefficients for manmade and natural channels. The range of Manning's n values covers from 0.012 for clean, regular, manmade channels to 0.15 for heavily wooded natural streams. These photographs are useful for a basic understanding of the range of Manning's coefficient for various sections of creeks in our environment. Chow [1959] also presented a method to compute the composite roughness coefficient when different areas of a channel cross-section have different roughness characteristics. Palmer [1954] from the U.S. Soil Conservation Service plotted Manning's n against \sqrt{R} for a variety of North American grass species. These curves are shown in Figure 7-14. The 5 classes A through E shown in the figure depend on the length and the "stand", i.e., the vigor and thickness of growth, of the grass according to the Table 7-1 below. For some of the reaches where the creek bottom and/or banks are covered with vegetation, these curves may be helpful.

**Table 7-1
Classification of Grass by Length and Stand**

Average Length of Grass (in)	Good Stand	Fair Stand
>30	A	B
11 – 24	B	C
6 – 10	C	D
2 – 6	D	D
< 2	E	E

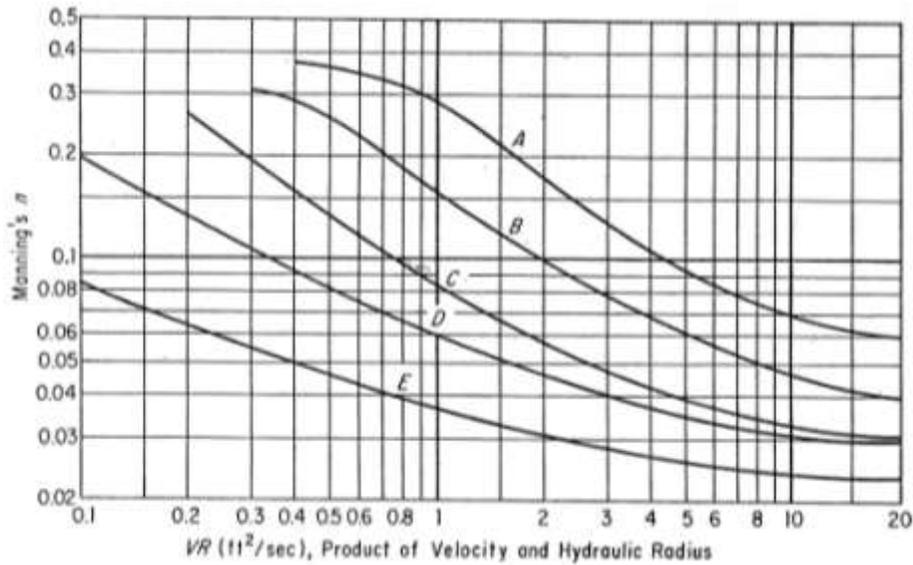


Figure 7-14. Manning's n in grassed channels

Henderson [1966] lists a table of Manning's coefficients for various artificial and natural materials. Barnes [1987] presents for various natural channels a photograph, a plan sketch, cross-sections and channel data to illustrate the corresponding Manning's coefficient. He covered channels where Manning's n ranges from 0.024 for wide, graveled rivers to 0.073 for mountainous or heavily wooded streams. These data appear to compare well with those shown in Chow [1959]. Arcement [1992] presented Manning's roughness coefficients for flood plains, where instead of earth, gravel or boulders, the areas are covered by wooded vegetation. The Manning's coefficient ranges from 0.1 to 0.2. The higher coefficients usually associate with heavy undergrowths. Arcement also provided a method, modified from that of Cowan [1956], for estimating channel and flood plain n values considering factors such as surface irregularities, geometry, obstructions and vegetation.

One last note on channel resistance is that both Chezy's and Manning's equations are applicable to any steady state flow conditions. If the flow is uniform, you may use simply the bed slope in the equations. If the flow is non-uniform, you will need to use the energy slope in the calculation.

7.6.1 Uniform Flow Calculations

As discussed earlier, when flow is uniform, there is no net force acting on a flow element in the channel. The result is that the bed slope, the water surface slope and the energy slope are all parallel, and flow characteristics remain the same from one section to another.

In reality, uniform flow seldom occurs in nature. However, uniform flow is a condition of such basic importance that it must be considered in all channel design problems. It is also used in many cases to obtain an easy and approximate solution to a non-uniform flow problem.

The Manning's equation is most commonly used to compute uniform flow characteristics. Given a flow rate, channel bottom slope, and estimated roughness coefficient n , the hydraulic radius may be calculated and the flow depth computed based on the cross-sectional geometry. This flow depth is called the *normal depth*, or *uniform depth*, often symbolized by y_o in the literature. The uniform flow condition is the criterion governing the most effective conveyance, i.e., requiring minimum cross-sectional area, for the given flow rate, bed slope and roughness coefficient.

It is worth noting that Chow [1959] provides charts to readily compute normal depth for circular and trapezoidal cross-sections, although these tasks may be easily handled now by computers.

7.6.2 Non-Uniform Flow Analysis

When the forces acting on a flow element is not balanced, and there is a resultant convective acceleration, the bed slope, water surface slope and total energy slope are not parallel any more. This is predominantly the case in the natural environment. As a hydraulic engineer, you will want to know how water depth and velocity behave in a channel before you finalize design modifications. The purpose of this section is to categorize flow conditions in a semi-quantitative manner so that we can predict flow behavior for design and analysis purposes. The first step is to classify channel bottom slopes.

Mild, Steep and Critical Slopes

A *mild* slope is one on which uniform flow is subcritical; on a *steep* slope uniform flow is supercritical; and on a *critical* slope uniform flow is critical. We can write

$$\begin{aligned}y_o &> y_c && \text{for mild slopes} \\y_o &< y_c && \text{for steep slopes} \\y_o &= y_c && \text{for critical slopes}\end{aligned}$$

Basic Non-Uniform Flow Equations

By the definition of energy slope, we have

$$\frac{dH}{dx} = \frac{d}{dx} \left(z + y + \frac{v^2}{2g} \right) = -S_f = -\frac{v^2}{C^2 R}$$

Hence,

$$\frac{d}{dx} \left(y + \frac{v^2}{2g} \right) = -\frac{dz}{dx} - S_f$$

i.e.,

$$\frac{dE}{dx} = S_o - S_f \tag{7-27}$$

In Section 7.4.3, we showed that

$$\frac{dE}{dy} = 1 - Fr^2 \tag{7-28}$$

Hence, Eq. (7-27) becomes

$$\frac{dy}{dx} (1 - Fr^2) = S_o - S_f \tag{7-29}$$

These are the general equations which will show us how the water surface changes with flow conditions.

Occurrence of Critical Flow

Consider a channel having a mild slope connecting at the downstream end to a steep slope, as is the case of Calabazas Creek at El Camino Real. This channel is schematized in Figure 7-15.

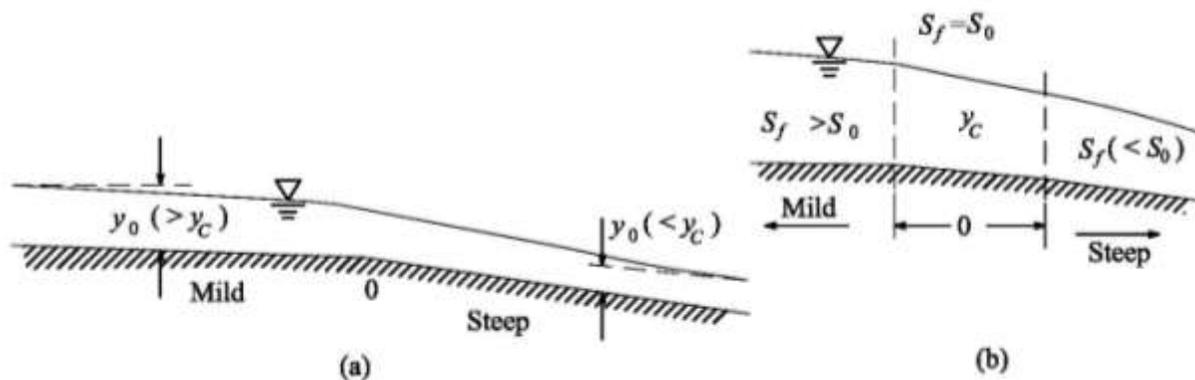


Figure 7-15. Critical flow at a change of slope

Let's examine Figure 7-15(a) first. On the mild slope upstream from the connection point O, the flow is subcritical. On the steep slope downstream from the connection, the flow is supercritical. Within the transition zone, the water depth keeps dropping from subcritical to become supercritical. Hence, immediately upstream from the connecting point O, we know that

$S_f > S_o$, and downstream from the connection point, $S_f < S_o$, as illustrated in Figure 7-15

(b). Hence, somewhere near the connecting point, $S_f = S_o$. From Eq. (7-29), we see that

when $S_f = S_o$, either

$$\frac{dy}{dx} = 0$$

or

$$Fr = 1$$

Since $\frac{dy}{dx}$ is clearly not zero in this neighborhood, it follows that Fr must be unity and the flow critical.

This example points out an interesting observation. When flow is released from a reservoir into a steep channel, it always goes through critical at or near the point of release.

Longitudinal Flow Profiles

The above example also illustrates that using Eq. (7-29) one may deduce various flow behaviors depending on the bed slope and flow condition at the moment. We can summarize these behaviors in the following paragraphs.

The flow behaviors are best understood when the flow domain is divided into 3 zones, as shown in Figure 7-16 for a channel on a mild slope.

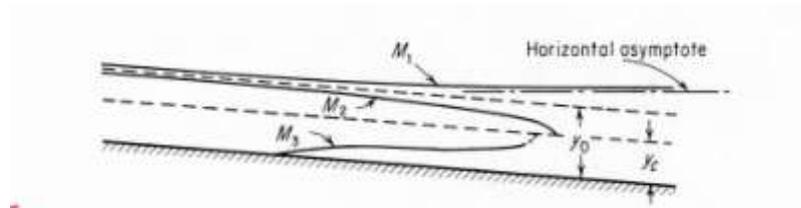


Figure 7-16. Longitudinal Flow Profiles on a Mild Slope

These profiles were deduced from the following relationships:

$$\begin{aligned} \text{for } y > y_o > y_c, \quad S_o > S_f, \quad Fr < 1, \quad \text{and } \frac{dy}{dx} > 0 \\ \text{for } y_o > y > y_c, \quad S_o < S_f, \quad Fr < 1, \quad \text{and } \frac{dy}{dx} < 0 \\ \text{for } y_o > y_c > y, \quad S_o < S_f, \quad Fr > 1, \quad \text{and } \frac{dy}{dx} > 0 \end{aligned}$$

Similar relationships may be obtained for other types of slopes, as shown in Figure 7-17.

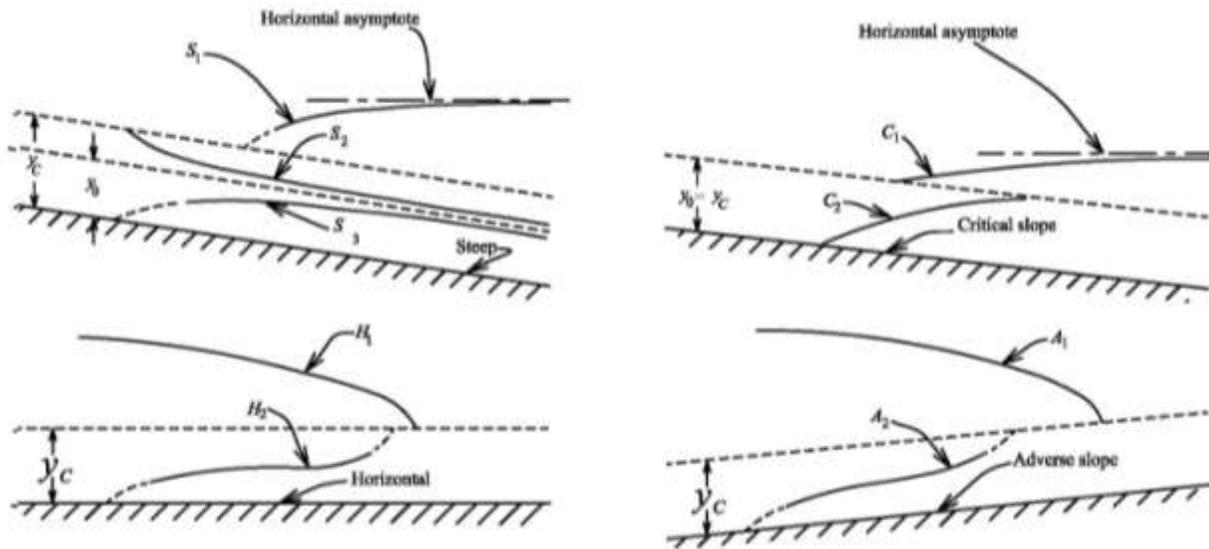


Figure 7-17. Longitudinal Profiles on Various Types of Slopes

7.7 CHANNEL CONTROLS AND TRANSITIONS

7.7.1 Concept of Channel Control

A control is a channel feature, natural or artificial, which fixes a relationship between depth and discharge in its neighborhood. Our interests in controls are twofold. First, the control allows us to design a channel to fulfill certain discharge requirements. The spillway of a dam is a typical example of this application. Secondly, it allows us to modify the flow regime or longitudinal profile to achieve certain channel protection functions. Drop structures installed to control channel grade are examples.

We also install gauge stations along our creeks to record water depth and flow rate. A flow rating curve, which is a relationship between depth and discharge, is developed for this purpose. Hence, the gauge station is a control. Some of these gauge stations are located in a natural section without any apparent channel modification work. For these stations, the rating curve needs to be developed based on our understanding of the flow regime and longitudinal profile of the creek, in addition to some limited field measurements. This is another reason why the knowledge of channel control is important.

In the following sections we will provide descriptions for a few channel controls that are typically used in our creeks and reservoirs.

7.7.2 Weirs, Spillways and Drop Structures

Sharp-Crested Weirs

A sharp-crested weir is shown in Figure 7-18.

Note that the pressure distribution at the crest of the weir is not hydrostatic due to the vertical acceleration of flow across this section. When the weir is long, and the effect of contraction from the sides may be ignored, the unit discharge may be estimated by

$$q = \frac{2}{3} C_d \sqrt{2g} H^{2/3} \quad (7-30)$$

where

$$C_d = 0.611 \quad \text{for } H/W = 0$$

$$C_d = 0.611 + 0.08 H/W \quad \text{for } H/W \leq 5$$

$$C_d = 1.06 \left(1 + \frac{W}{H} \right)^{3/2} \quad \text{for } 5 < H/W < 10$$

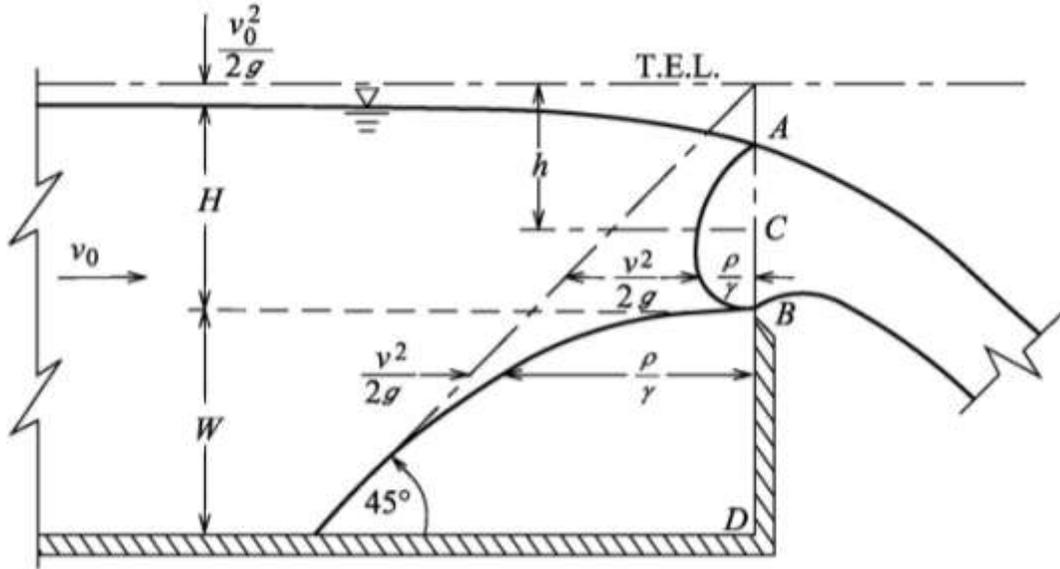


Figure 7-18. Sharp-Crested Weir

Note that when $W = 0$ the situation approaches that of a free overfall, and $C_d = 1.06$.

When the side or bottom contraction cannot be ignored, as reflected in Figure 7-19, the unit discharge concept is not applicable anymore and the discharge equation becomes

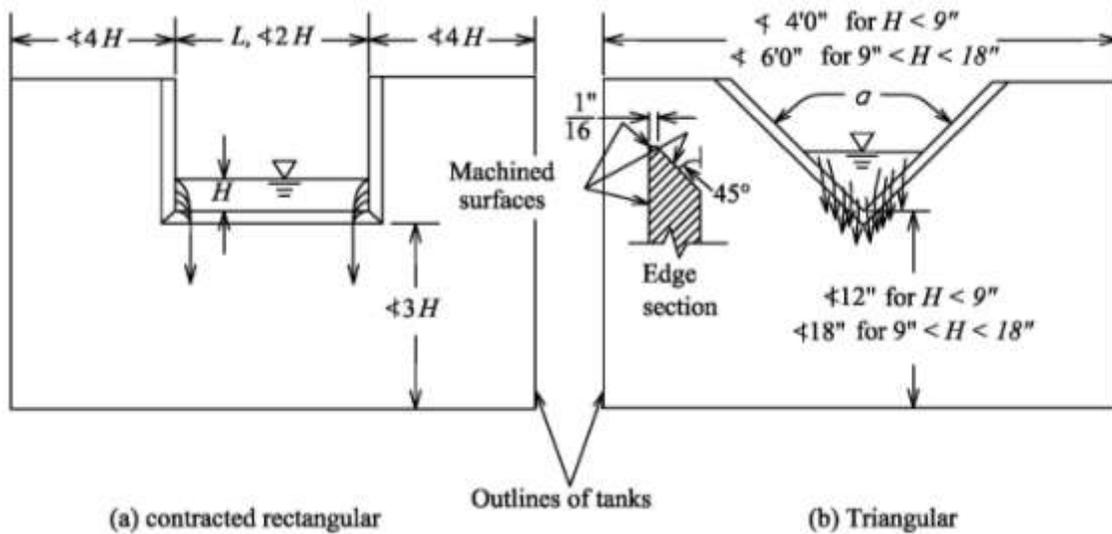


Figure 7-19. Typical Designs of Rectangular and Triangular Weirs

$$Q = \frac{2}{3} C_c (L - 0.2H) \sqrt{2g} H^{3/2} \quad (7-31)$$

for rectangular weirs, where $C_c = 0.611$. For triangular weirs,

$$Q = \frac{8}{15} C_c \tan \frac{\alpha}{2} \sqrt{2g} H^{5/2} \quad (7-32)$$

And when $\alpha = 90^\circ$, $C_c = 0.585$, and the discharge equation becomes

$$Q = 2.5 H^{5/2} \quad (7-33)$$

Broad-Crested Weir

Figure 7-20 illustrates three weirs of various lengths. For most cases where there is no downstream backwater effect and the weir is not longer than 3 times of H, the weir discharge may be estimated by measuring the brink depth y_b and relating it to the critical depth as follows:

$$q = 1.65 y_b \sqrt{g y_b} \quad (7-34)$$

where

$$y_b = 0.715 y_c$$

When the weir is longer than 3H, the effect of bottom boundary layer should be considered. Refer to p.212 of Henderson [1966] for reference.

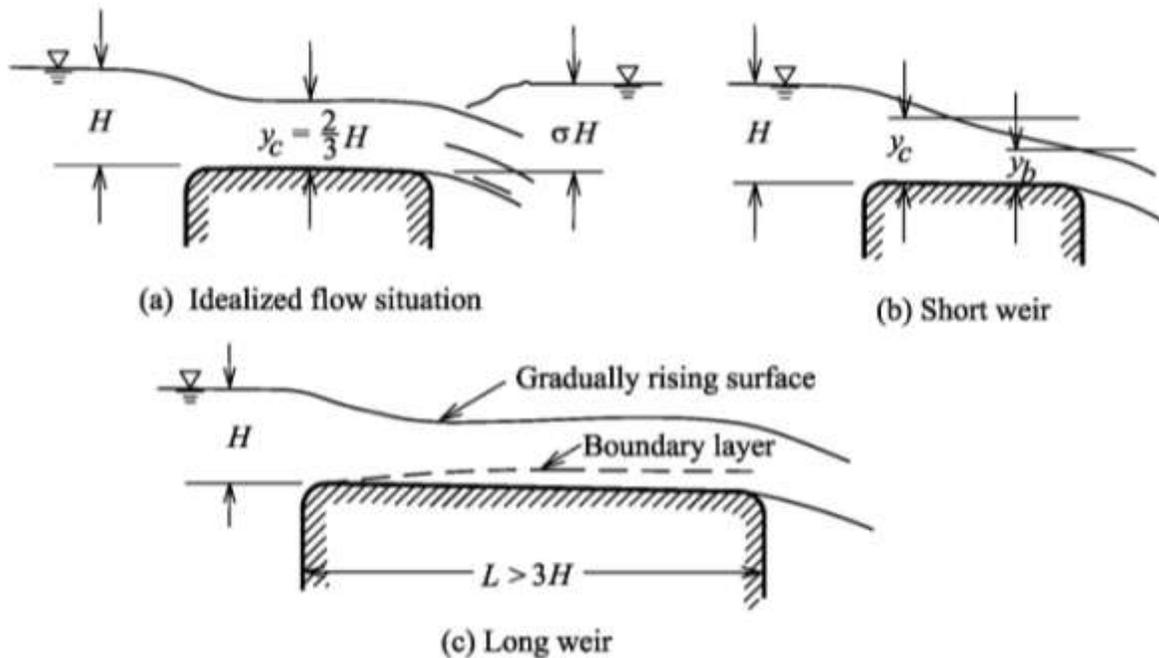


Figure 7-20. Broad-Crested Weirs

If there is downstream backwater effect, such as shown in Figure 7-20(a), with the submergence factor σ greater than 0.83 – 0.85, the discharge equation will no longer be applicable. In design, use a conservative number of 0.80 to determine if backwater effect exists. Below this value the weir will discharge freely, with no submergence effects.

Drop Structure

We discussed using the brim depth to compute discharges over broad-crested weirs. The same also applies to drop structures.

As shown in Figure 7-21, without downstream submergence effect, the pressure distribution at the brink of the drop is significantly deviated from hydrostatic. Measurements of laboratory experiments show that at a distance of 3 to 4 times the depth upstream from the brink the flow is critical. Additionally, the vertical acceleration becomes insignificant at this point and the pressure returns to hydrostatic. Hence, if the brim depth is measured, the unit discharge becomes

$$q = y_c \sqrt{g y_c} = \frac{\sqrt{g}}{0.6} y_b^{3/2} \quad (7-35)$$

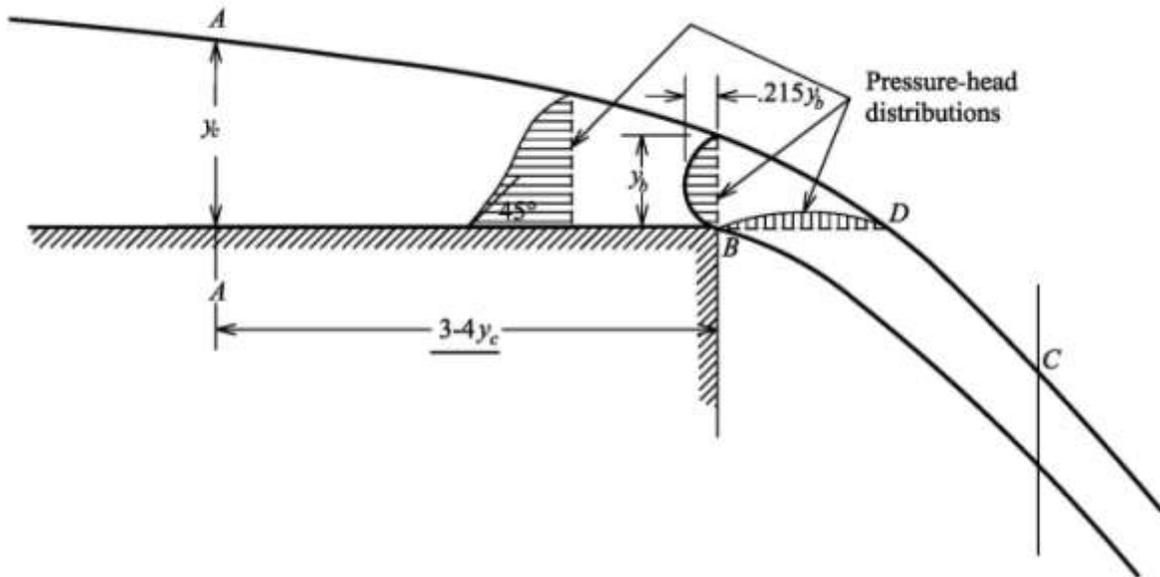


Figure 7-21. Flow Over a Drop Structure

Many researchers have made measurements of brink depth in circular pipes running full. The empirical discharge vs. brink depth relationship is

$$\frac{Q}{D^2 \sqrt{gD}} = 1.55 \left(\frac{y_b}{D} \right)^{1.88} \quad (7-36)$$

7.7.3 Hydraulic Jump

We discussed earlier in Section 7.5.2 that across a hydraulic jump the upstream and downstream depths are related and are called the conjugate depths. We now can show how these depths are related. For a simple hydraulic jump, $P_f = 0$, and Eq. (7-15) becomes

$$\frac{q^2}{g} \left(\frac{1}{y_1} - \frac{1}{y_2} \right) = \frac{1}{2} (y_2^2 - y_1^2)$$

and then

$$Fr_1^2 = \frac{1}{2} \frac{y_2}{y_1} \left(\frac{y_2}{y_1} + 1 \right) \quad (7-37)$$

This is the well-known equation of the hydraulic jump. If the upstream condition is known, the relationship between upstream and downstream depths may be expressed by

$$\frac{y_2}{y_1} = \frac{1}{2} \left(\sqrt{1 + 8 Fr_1^2} - 1 \right) \quad (7-38)$$

On the other hand, if the downstream condition is known, the relationship may be written as

$$\frac{y_1}{y_2} = \frac{1}{2} \left(\sqrt{1 + 8 Fr_2^2} - 1 \right) \quad (7-39)$$

Also important is the energy loss across the jump. This loss may be expressed based on the principle of conservation of energy, once the characteristics of the jump are computed, as

$$\Delta E = \frac{y_2^3 - y_1^3}{4 y_1 y_2} \quad (7-40)$$

Another characteristic is the length of jump. It depends on the slope of the channel and tailwater elevation. Because of the variability, the coverage is lengthy and is not included here. Refer to p. 218 of Henderson [1966] if you need the data for calculation.

Figures 7-22 and 7-23 show 2 useful applications where a hydraulic jump is induced by either a channel bottom rise or drop.

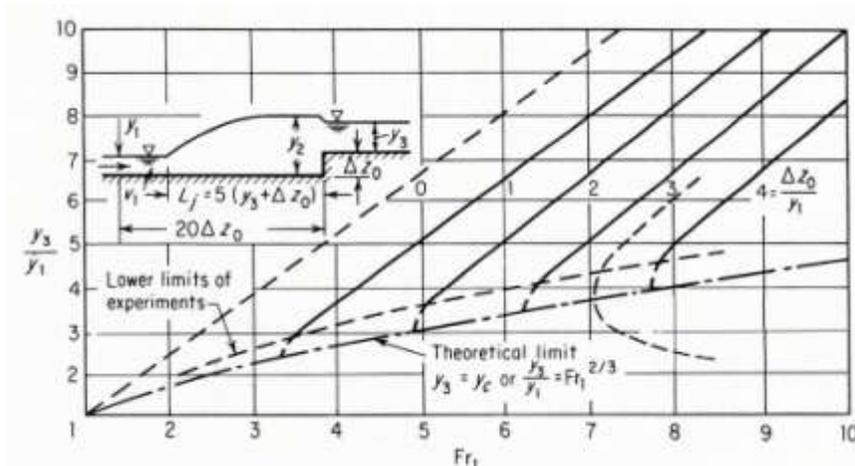


Figure 7-22. Hydraulic Jump at an Abrupt Rise, After J. W. Forster and R. A. Skrinde [1950]

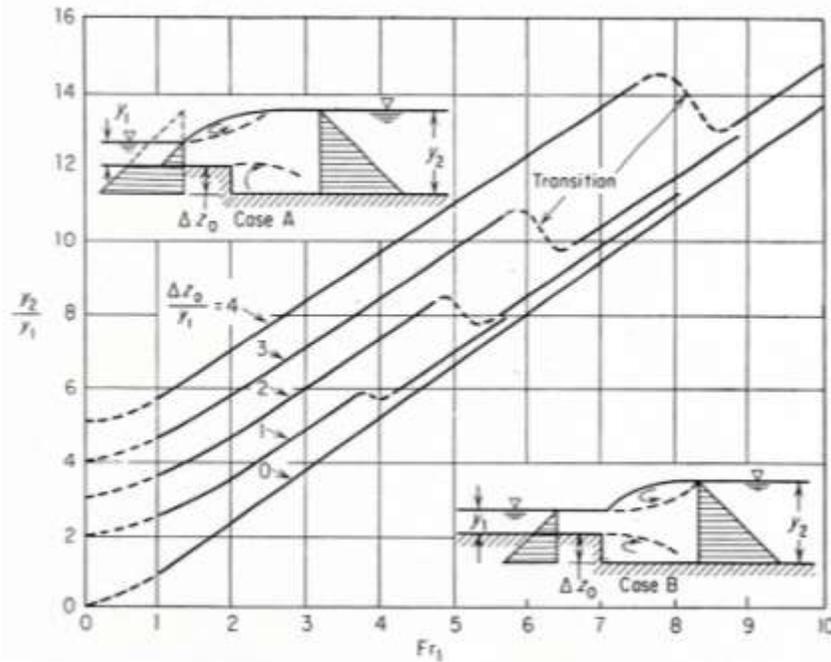


Figure 7-23. Hydraulic jump at an abrupt drop, after E. Y. Hsu [1950]

Depending on the upstream flow condition and the channel roughness, various forms of hydraulic jump and different levels of turbulence will result. Through this process, air also is entrained into the jump to cause boil on the surface. Hence, sufficient freeboard should be provided in design. For the St. Anthony Falls type of stilling basin that is usually designed to contain the jump, a freeboard of 1/3 of the downstream water depth is specified.

7.7.4 Expansions and Contractions

Channel transitions such as expansion and contraction normally may be analyzed using principles of conservation of energy and momentum for sub-critical flows. For supercritical flows, the channel alignment changes will generate shock waves and present a complication that needs to be dealt with separately.

Sub-Critical Flow

As shown in Figure 7-24, the approach to analyze this expansion problem is to set $E_1 = E_2$ and $M_2 = M_3$. Then the energy loss through this sudden expansion may be derived to be equal to

$$E_1 - E_3 = \Delta E = \frac{v_1^2}{2g} \left[\left(1 - \frac{b_1}{b_2} \right)^2 + \frac{2Fr_1^2 b_1^3 (b_2 - b_1)}{b_2^4} \right] \quad (7-41)$$

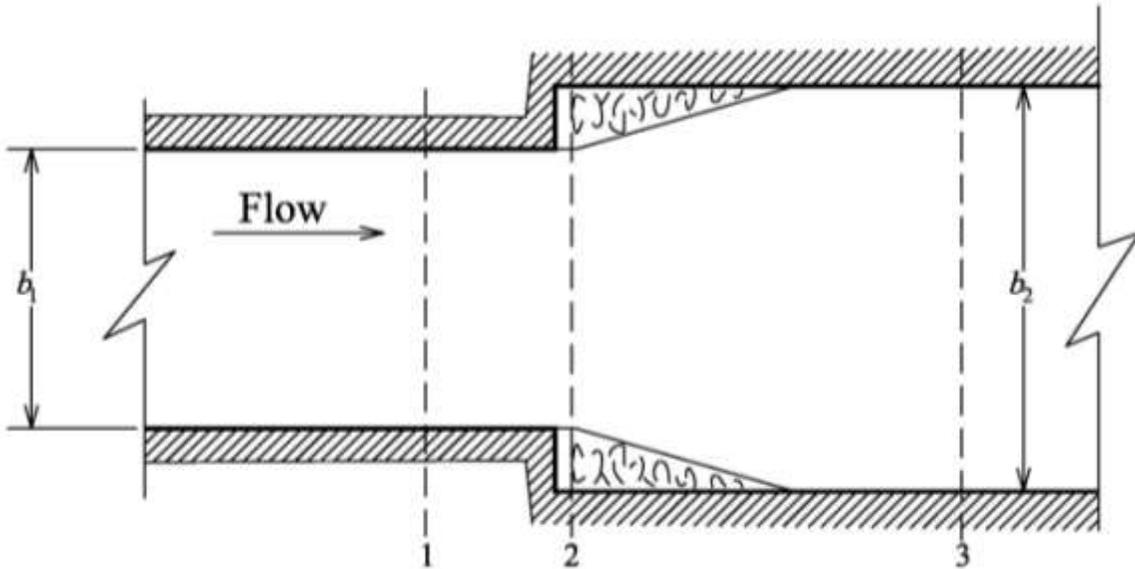


Figure 7-24. Sudden Expansion

The first term in the bracket is the same as that for a closed conduit, and the second term is for open channel flow. Laboratory experiments have shown that when tapering the expansion angle to a slope of 1 transversal to 4 longitudinal, the expansion loss drops to less than 1/3 from that of a sudden expansion. The loss is

$$\begin{aligned} \Delta E &= 0.3 \frac{v_1^2 - v_3^2}{2g} \\ &= 0.1 \left(\frac{v_1^2}{2g} - \frac{v_3^2}{2g} \right) \\ &= 0.1 \Delta h_v \end{aligned}$$

where Δh_v is the difference in velocity head between the upstream and downstream. Further tapering will be less effective in reducing head loss compared to the increase in cost. Hence, a rule of thumb for design is to limit the rate of expansion to 1:4, or approximately at an angle of 14°.

Channel contraction usually incurs less head loss than expansion, simply because as flow is accelerated there is less waste in energy due to eddies and other ineffective flow patterns. The following Table 7-2, excerpted from p. 26 of USACE [1995], lists head loss coefficients for different types of channel expansion and contraction that may be used for design.

**Table 7-2
Transition Loss Coefficient**

Transition Type	C_c	C_e	Source of Data
Warped	0.10	0.20	Chow [1959] Brater & King [1976]
Cylindrical Quadrant	0.15	0.20	Chow [1959]
Wedge	0.30	0.50	USBR [1967]
Straight Line	0.30	0.50	Chow [1959]
Square End	0.30	0.75	Chow [1959]

Note: Contraction Loss = $h_c = C_c \Delta h_v$

Expansion Loss = $h_e = C_e \Delta h_v$

Super-Critical Flow

Channel expansion and contraction in supercritical flow will generate cross waves. These waves necessitate higher channel walls to contain the flow. Experiments by Ippen and Dawson [1951] have found that straight contractions are always better than curved contractions of equal length of contraction from the standpoint of maximum height. Accordingly, they have proposed a procedure of design for straight contractions. Rouse et al. [1951], on the other hand, developed a procedure to design curved channel expansions to minimize wave disturbances. These procedures may be found in both Chow [1959] and Henderson [1966], and are not included here.

It suffices to know that there will be additional requirements on wall height due to cross waves generated in supercritical flows. Should the need arise to design a channel in such flow conditions, a physical model study should be considered to determine the head loss through the transition and proper wall height for the expected range of flows.

7.7.5 Channel Bend

There are two subjects related to channel bends that a hydraulic engineer should be concerned about. First is superelevation. The centrifugal force generated by flow around a curve will cause the water surface to rise along the outside wall and depress along the inside wall. The rise of water surface over the mean water depth is called superelevation.

Secondly, the transverse water surface gradient, caused by superelevation, will generate secondary currents which may propagate a long distance downstream. Because the velocity closer to the bottom is slower, the transverse pressure gradient thus generates an inward flow near the bottom. This in turn generates an outward flow in the upper levels, resulting in a circulatory motion in the cross-section. This secondary current tends to scour sediment from the outside of the bend and deposit it on the inside, forming shallow point bars on the inside and deep pools on the outside.

The scouring mechanism of the secondary current then indicates that if you are to provide a branch channel off a creek, take it off the outside of a bend will result in much less entrained sediment in the branch channel. This fact was realized even in ancient times. A study of the irrigation systems built by ancient Mediterranean civilizations show that branch channels were always connected either to straight sections or to the outside of curves of a main channel.

Other than understanding the flow behavior stated above, a hydraulic engineer may want to design a channel bend such that the discharge remains approximately uniform across the width. This will lead in a dropped bottom for supercritical flow or raised bottom for subcritical flow in the inside of the bend. Figure 7-25 provides a definition sketch for the bottom profile of a bend.

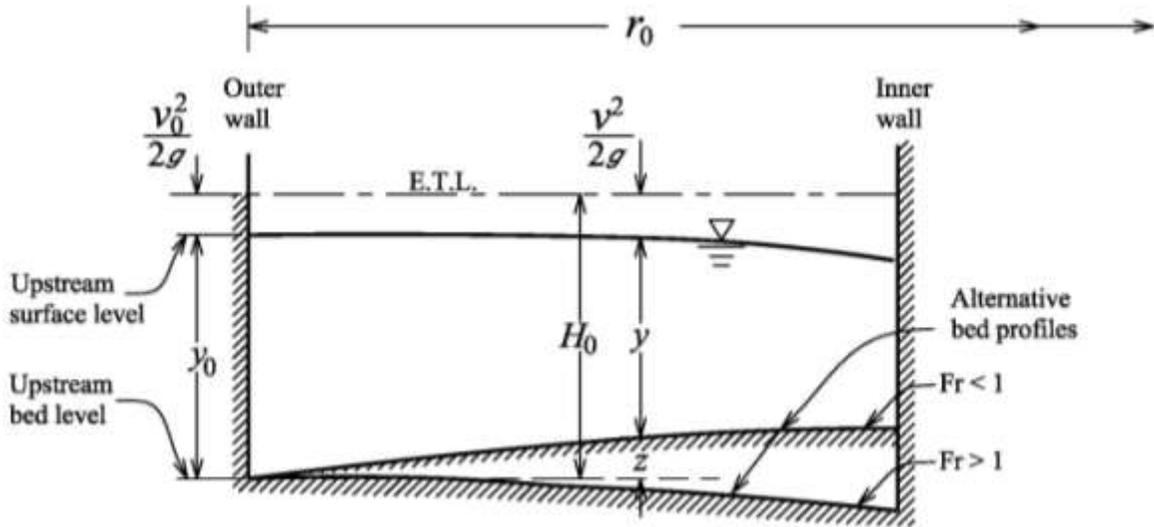


Figure 7-25. Transverse Bed Profiles Required at a Channel Bend for Constant q

$$z + \frac{qr}{v_o r_o} + \frac{v_o^2 r_o^2}{2g r^2} = H_o \quad (7-42)$$

Eq. (7-42) defines the transverse bed profile required to obtain a constant q across the section. Knapp and Ippen [1939] proposed to add transition curves of radius $2r_c$ upstream and downstream of the main curve of radius r_c . The length of the transition curves is obtained by Eq. (7-43) as shown in Figure 7-26.

Table 7-3
Short and Steep Box Culvert Discharge Relationships

H/D	Discharge Equation	Notes
<1.2	$Q = \frac{2}{3} C_B B H \sqrt{\frac{2}{3} g H}$ <p align="right">Eq. (7-44)</p>	Water surface below culvert soffit. Flow critical at inlet. B is width. C_B represents coefficient for contraction from side walls, equal to 1 for rounded edges to a radius of 0.1B, equal to 0.9 for square edges.
>1.2	$Q = C_h B D \sqrt{2g(H - C_h D)}$ <p align="right">Eq. (7-45)</p>	Water surface touches the soffit. C_h represents the effect of contraction from top and side walls, equal to 0.8 for rounded edges and 0.6 for square edges.

Outlet Control

The above equations apply if the culvert runs open-channel, and is under inlet control. When the tailwater level rises to cause the culvert to run full and outlet control takes over, compute the discharge by taking an energy balance between the upstream and downstream water levels and estimate losses incurred through the entrance, culvert friction and outlet expansion. U.S. Bureau of Reclamation [1974] provides the following entrance loss coefficients in Table 7-4 based on experiments. These loss coefficients are referenced to the average velocity in the culvert, i.e.,

$$H_L = K_e \frac{v^2}{2g}$$

where H_L is the head loss, K_e the entrance loss coefficient, and v the average velocity in the culvert.

**Table 7-4
Culvert Entrance Loss Coefficients**

Entrance Condition	Range of Loss Coefficient (K_e)	Average Loss Coefficient (K_e)
Square-edged inlets installed flush with vertical headwalls	0.43 – 0.70	0.50
Rounded inlets installed flush with vertical headwalls, $r/D \geq 0.15$	0.08 – 0.27	0.10
Grooved or socket-ended concrete pipe installed flush with vertical headwalls	0.10 – 0.33	0.15
Projecting concrete pipe with grooved or socket ends		0.20
Projecting steel or corrugated metal pipes	0.50 – 0.90	0.85

The culvert outlet loss may be estimated by multiplying the appropriate expansion loss coefficient (C_e) discussed in Table 7-2 to the difference in velocity heads between the culvert and the outlet channel.

7.7.7 Bridge Piers

The subject of bridge piers is discussed here for 2 purposes. First, the piers reduce the local channel width. Based on what we learned in the E-y curves, for subcritical flows they will cause the water level to drop, and in some cases force the flow to go through critical and induce a hydraulic jump. Hence, a hydraulic engineer should always check the hydraulics of bridge crossings in design and flood routing. Secondly, bridge piers generate head loss and local scour. These impacts should be determined during design as well.

For most cases, the long axis of the bridge piers should be aligned with the direction of flow to minimize turbulence and head losses. If the bridge and flow are not completely aligned, the effective width of the bridge should be calculated by projecting the piers normal to the flow. This method may apply for piers skewed at no more than 20° from the direction of flow.

Because there will be drag forces acting on the surface of the piers, one should use the momentum equation with estimates of drag forces to compute the hydraulics at the pier location. There has been extensive research done on drag forces on cylinders, rectangular plates, and different shapes of airfoils in the literature. Defining a drag coefficient by

$$C_D = \frac{\text{drag force}}{A\rho V^2/2}$$

For most piers with slightly streamlined upstream and downstream faces, the drag coefficient may be estimated to be 1.2 [Rouse, 1978]. Given the surface area of the pier to be water depth multiplied by the circumference, the drag force on the pier may be computed and then substituted for P_f in the momentum Equation (7-15) to compute the bridge hydraulics. This may serve as an independent check on the bridge hydraulics of standard programs such as HEC-RAS [USACE 2006].

As for local scour generated at bridge piers, more discussion will be provided in Chapter 2.

7.8 UNSTEADY FLOW

Because the sizes of our watersheds in the Santa Clara Valley are relatively small, and the time of concentration of flood water is relatively short, floods happen in a flashy mode. Flood water comes swiftly and recedes just as swiftly, except for those flood plain areas plugged up by debris. Hence, temporary flood control measures, such as retention pond and closure structure, may significantly affect flood intensity and should be included in the design considerations. This makes the unsteady flow simulation a necessary tool for our projects.

7.8.1 Unsteady Water Surface Profile Computation

The equation of continuity for unsteady flow was introduced as Eq. (7-1)

$$\frac{\partial Q}{\partial x} + B \frac{\partial y}{\partial t} = 0 \quad (7-1)$$

Alternatively this equation may be written as

$$A \frac{\partial v}{\partial x} + v \frac{\partial A}{\partial x} + B \frac{\partial y}{\partial t} = 0 \quad (7-44)$$

On the other hand, the equation of motion may be written as

$$S_f = \frac{V^2}{C^2 R} = S_o - \frac{\partial y}{\partial x} - \frac{v}{g} \frac{\partial v}{\partial x} - \frac{1}{g} \frac{\partial v}{\partial t} \quad (7-45)$$

where C is the Chezy C and R the hydraulic radius. This is the general equation of motion applicable to steady or unsteady and uniform or non-uniform flows. Both the continuity and momentum equations (7-44) and (7-45) were first published by Saint-Venant in 1848. The task now is to solve these equations for the two unknowns v and y . Owing to their mathematical complexity, exact integration is practically impossible. There are various graphical, semi-graphical, and numerical methods developed to derive a solution. HEC-RAS [USACE 2006] adopts an implicit finite difference scheme to solve the set of one dimensional unsteady flow equations. FESWMS [FHWA 2002] uses a finite element method to solve a set of two dimensional equations. These numerical models are adequate to address most of the unsteady flow problems that we have. It appears that an engineer does not need to know the detailed solution technique to apply these models. Hence, we will not discuss further on the subject. However, it is still important to have the basic knowledge of the unsteady flow characteristics such as positive and negative surge formations, tidal bore, wave celerity, etc. that appear in our surroundings. Chapter 3 of Henderson [1966] gave a detailed description of these phenomena.

Also it should be mentioned that the calculation of unsteady flow profiles becomes important when we need to determine the storage requirements of a retention pond to limit peak flood discharges. A typical flood hydrograph of our watersheds has a narrow but sharp rise to the peak discharge. It therefore does not require much storage volume to effectively reduce the peak discharge. Routing such a flood hydrograph using an unsteady flood routing program, such as HEC-RAS, will enable optimal design of the retention facility.

BIBLIOGRAPHY

- Arcement, Jr., G. J., and V. R. Schneider, Guide for Selecting Manning's Roughness coefficients for Natural Channels and Flood Plains, *U.S. Geological Survey Water-Supply Paper 2339*, U.S. Government Printing Office, 1992.
- ASCE (American Society of Civil Engineers), Task Force Report on Friction Factors in Open Channels, *Proc. Am. Soc. Civil Engineers*, Vol. 89, No. HY2, March 1963.
- Barnes, Jr., Harry H., Roughness Characteristics of Natural Channels, *U.S. Geological Survey Water Supply Paper 1849*, U.S. Government Printing Office, Washington, 1987.
- Brater, E. F. and H. W. King, *Handbook of Hydraulics for the Solution of Hydraulic Engineering Problems*, 6th edition, McGraw-Hill, New York, 1976.
- Bresse, J. A. Ch., Course in Applied Mechanics, Mallet-Bachelier, Paris, 1860.
- Chow, V. T., *Open-Channel Hydraulics*, McGraw-Hill, New York, 1959.
- Cowan, W. L., "Estimating Hydraulic Roughness Coefficients," *Agricultural Engineering*, Vol. 37, No. 7, pp. 473 – 475, July, 1956.
- Dunne, T. and L. B. Leopold, *Water in Environmental Planning*, W. H. Freeman Co., San Francisco, 1978.
- FHWA, Federal Highway Administration, *User's Manual for FESWMS Flo2DH*, Two-dimensional depth-averaged flow and sediment transport model, Release 3, Publication No. FHWA-RD-03-053, September 2002.
- Forster, J. W. and R. A. Skrinde, "Control of the Hydraulic Jump by Sills," *Trans. Am. Soc. Civil Engineers*, Vol. 115, 1950.
- Henderson, F. M., *Open Channel Flow*, The Macmillan Company, New York, 1966.
- Hsu, E. Y., discussion on "Control of the Hydraulic Jump by Sills" by Forster, J. W. and R. A. Skrinde, *Trans. Am. Soc. Civil Engineers*, Vol. 115, p. 988, 1950.
- Ippen, A. T. and J. H. Dawson, "Design of Channel Contractions," 3rd paper in Symposium on High-Velocity Flow in Open Channels, Transactions, *American Society of Civil Engineers*, Vol. 116, pp. 326-346, 1951.
- Knapp, R. T. and A. T. Ippen, "Curvilinear Flow of Liquids With Free Surface at Velocities Above That Of Wave Propagation," Proceedings, *5th International Congress of Applied Mechanics*, New York, John Wiley & Sons, Inc., 1939.
- Leopold, L. B. and M. G. Wolman, "River Channel Patterns: Braided, Meandering, and Straight," *U.S. Geological Survey Professional Paper 282-B*, U.S. Government Printing Office, Washington, DC, 1957.

- Lane, E. W., "A Study of the Shape of Channels Formed by Natural Streams Flowing in Erodible Material," *Missouri River Division Sediment Series No. 9*, U.S. Army Engineer Division, Missouri River, Corps of Engineers, Omaha, Nebraska, 1957.
- Mollard, J. D. and J. R. Janes, "Airphoto Interpretation and the Canadian Landscape," Department of Energy, Mines and Resources, Ottawa, Canada, Canadian Government Publishing Center, Supply and Services, Hull, Quebec, Canada, 1984.
- North Carolina Urban Water Consortium, "How do you Identify an Intermittent Stream?" 2000-2001 Annual Program, Water Resources Research Institute of the University of North Carolina, 2001.
- Palmer, V. J. and W. O. Ree, "Handbook of Channel Design for Soil and Water Conservation," *U.S. Department of Agriculture, SCS-TP-61*, 1954.
- Rosgen, Dave, *Applied River Morphology*, second edition, Wildland Hydrology, Pagosa Springs, Colorado, 1996.
- Rouse, Hunter, *Elementary Mechanics of Fluids*, Dover Publications, Inc., New York, 1946.
- Rouse, H., B. V. Bhoota and E. Y. Hsu, "Design of Channel Expansions," 4th paper in Symposium of High-velocity flow in open channels, *Transactions*, American Society of Civil Engineers, Vol. 116, pp. 347-367, 1951.
- Schumm, S. A. and D. F. Meyer, "Morphology of Alluvial Rivers of the Great Plains," Riparian and Wetlands Habitats of the Great Plains, *Great Plains Agricultural Council Publication 91*, pp. 9-14, 1979.
- U.S. Army Corps of Engineers, HEC-RAS River Analysis System, *User's Manual*, Version 4.0 Beta, CPD-68, Hydrologic Engineering Center, Davis, CA., November 2006.
- U.S. Army Corps of Engineers, CE-QUAL-W2: A Two-Dimensional Laterally Averaged Hydrodynamic and Water Quality Model, Version 3.1, *User Manual*, Instruction Report EL-03-1, Washington DC, 20314-1000, 2003.
- U.S. Army Corps of Engineers, Hydraulic Design of Flood Control Channels, EM 1110-2-1601, 1995.
- U.S. Army Corps of Engineers, Channel Stability Assessment for Flood Control Projects, EM 1110-2-1418, October 1994.
- U.S. Army Corps of Engineers, *River Hydraulics*, EM 1110-2-1416, 1993.
- U.S. Bureau of Reclamation, Design of Small Dams, Section 213 Culvert Spillways, pp. 430 – 439, U.S. Department of the Interior, Denver, CO, 1974.
- U.S. Bureau of Reclamation, General Design Information for Structures, Chapter 2, Canals and Related Structures, Design Standards No. 3, U.S. Department of the Interior, Denver, CO, 1967.